University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 3 Multiple Choice

Sections Covered: 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.7, 4.8

Read This First!

- These practice problems are NOT sufficient review for the exam and do not represent the exact length of the exam. You should also use other resources, such as the textbook, worksheets, and Paul's Online notes to find further practice problems on topics that you have struggled with (or that you have trouble with on the practice problem set).

- Use these practice problems, in addition to other course materials, as a guide to determine what you need to study more deeply.

- The exam will contain some multiple choice questions as well as short-answer questions. **Short answer questions may be similar to questions found in lecture videos, live class activities, worksheets, and/or WebAsisgn.** When studying, make sure you are able to **fully justify your answers and reasoning** to prepare for the short-answer portion of the exam.

- The exam will be 50 minutes during your regular discussion section meeting.

- Please read each question carefully. For multiple choice questions, there is only one correct choice for each answer.

- On the exam, please carefully check all of your final answers. The submitted letter answers are the **ONLY** place that counts as your official answers for multiple-choice questions.

- You may **NOT** use a calculator or any other references on the exam, and **you are expected to work independently.**

1. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ (C) $\frac{17}{8}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

$f(1.1) \approx f(1) + (1.1-1) \cdot f'(1)$

$f(1) = \sqrt{1+2+1} = 2$

$f'(x) = \frac{1}{2\sqrt{x^3+2x+1}} (3x^2+2)$

$f'(1) = \frac{1}{2 \cdot 2} \cdot 5 = \frac{5}{4}$

$f(1.1) \approx 2 + 0.1 \cdot \frac{5}{4} = 2 + \frac{5}{40} = 2 + \frac{1}{8} = \frac{17}{8}$

2. Which of the following is the absolute maximum value of the function $f(x) = \frac{x}{x^2+4}$ on the interval $\boxed{[0, 4]}$?

(A) $\frac{1}{8}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$

(D) $\frac{1}{2}$ (E) 1

$f'(x) = \frac{x^2+4 - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$

$f'(x) = 0$ only when $4 = x^2$, $x = \pm 2$, only $+2$ is in $[0,4]$

$f(x)$ differentiable on $(0, 4)$

Only points where absolute max can be are $x = 2$ or the endpoints $0, 4$

x	$f(x)$
0	0
2	$\frac{2}{2^2+4} = \frac{1}{4}$ largest
4	$\frac{4}{4^2+4} = \frac{4}{20} = \frac{1}{5}$

3. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(\overset{x}{2}, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

$$\frac{dy}{dt} = \frac{1}{3\sqrt[3]{(x^4+11)^2}} \cdot 4x^3 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{4} \frac{\sqrt[3]{(x^4+11)^2}}{x^3} \cdot \underbrace{\frac{dy}{dt}}_{32}$$

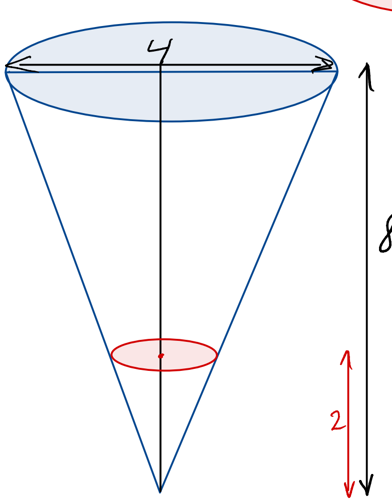
$$= \frac{3}{4} \cdot \frac{\sqrt[3]{(2^4+11)^2}}{2^3} \cdot 32$$

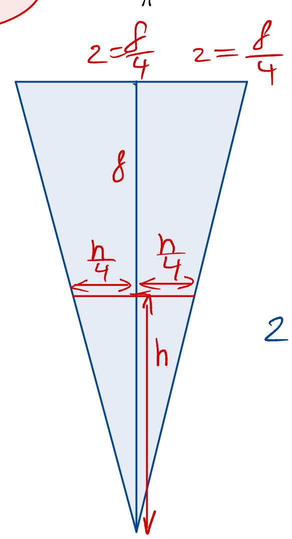
$$= 3\sqrt[3]{27^2} = 3 \cdot 3^2 = 27$$

4. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3}r^2h$?)

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min





$$r(h) = \frac{h}{4}$$

$$V(h) = \frac{\pi}{3} \frac{h^2}{16} \cdot h = \frac{\pi}{3} \frac{h^3}{16}$$

$$\underbrace{\frac{dV}{dt}}_{2\ ft^3/min} = \frac{\pi}{16} \underbrace{h^2}_{2^2} \frac{dh}{dt}$$

$$\frac{dh}{dt} = 2 \cdot \frac{16}{4\pi} = \frac{8}{\pi}$$

5. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

(A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ (B) $x_{n+1} = (x_n^2 + 7)/(2x_n)$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

(D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x^2 - 7$

$f'(x) = 2x$

$x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n} = \frac{2x_n^2 - (x_n^2 - 7)}{2x_n} = \frac{x_n^2 + 7}{2x_n}$

6. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 (B) 19/6 (C) 15/4

(D) 12/7 (E) 17/6

$f(x) = x^2 - 10$

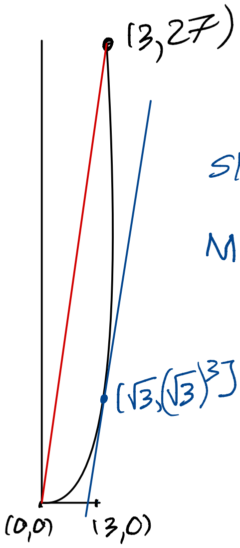
$f'(x) = 2x$

$x_{n+1} = x_n - \frac{x_n^2 - 10}{2x_n}$

$x_1 = 3$

$x_2 = 3 - \frac{3^2 - 10}{2 \cdot 3} = 3 - \frac{-1}{6} = 3 + \frac{1}{6} = \frac{19}{6}$

7. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.



(A) 9 (B) $\sqrt{27}$ (C) $\sqrt{3}$

(D) 3 (E) No such value of c exists.

Slope on $[0,3]$ is $\frac{27-0}{3-0} = 9$

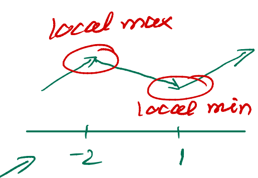
M.V.T. wants c in $[0,3]$ $f'(c) = 9$, $f'(c) = 3c^2$

$c^2 = 3$ $c = \pm\sqrt{3}$ only $\sqrt{3}$ in $[0,3]$

8. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

(A) 1 (B) -2 (C) $-2,\ 1$

(D) $-2,\ \frac{1}{2}$ (E) $-2,\ \frac{1}{2},\ 1$



$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$

$f'(x) = 0$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$

Critical numbers $x = -2,\ x = 1$

With 1st derivative test

$f'(x)$: $+$ (neg. · neg., $x+2$, $x-1$) -2 $-$ (pos. · neg., $x+2$, $x-1$) 1 $+$ (pos · pos, $x+2$, $x-1$) x

With 2nd derivative test

$f''(x) = 6(2x+1)$

$f''(-2) = -18$ local max

$f''(1) = +18$ local min

9. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16$

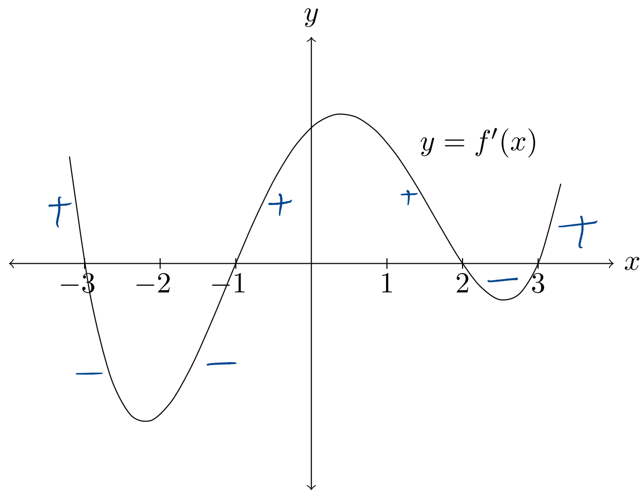
$f''(x) = 12\left(x^2 - \frac{16}{12}\right) = 12\left(x^2 - \frac{4}{3}\right)$

$f''(x) = 0$: $x = \pm\sqrt{\frac{4}{3}}$

f'' switches from $+$ to $-$ at $x = -\sqrt{\frac{4}{3}}$ and $-$ to $+$ at $x = \sqrt{\frac{4}{3}}$

So $\pm\sqrt{\frac{4}{3}}$ are both inflection points

10. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



x		-3		-1		2		3	
$f'(x)$	$+$	0	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$		$\nearrow$
		local max		local min		local max		local min	

(A) Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

11. Referring to the same graph of the derivative in the question above, at approximately what x-value(s) is $f(x)$ concave up?

(A) $x < -1$ and $x > 1.5$

(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

between these $f''(x) > 0$ so concave up

after this $f''(x) > 0$

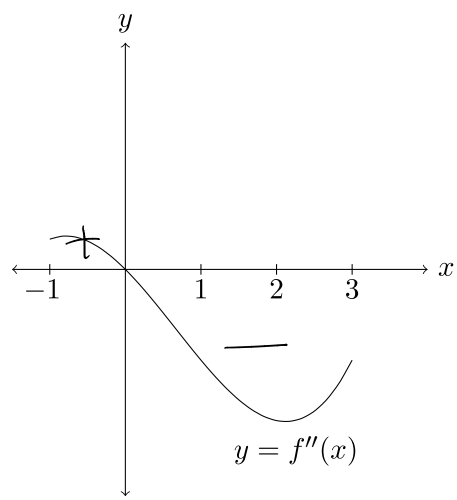
(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

$f''(x) = 0$ for $x \approx -2.1$, $x \approx 0.8$, $x \approx 2.6$ where we see humps in the graph of $f'(x)$. so concave up again

$f'(x)$ decreases, then increases, then decreases, then increases again.
$f''(x)$ $-$, then $+$, then $-$, then $+$

12. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. ✓ $f'' > 0$

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. ✓ $f'' = (f')' < 0$ on $(0,3)$

(C) The function $f(x)$ has a point of inflection at $x = 0$. ✓ f'' goes $+ \to -$

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. ✓ f' ↗ ↘ -1 0 3

(E) All of the above.

13. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x) = 12(x-2)(x-1)$

$+ \quad - \quad +$

$1 \quad 2$

14. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}.$$

$\sin x \to 0$, $x^2 \to 0$: indeterminate form $\frac{0}{0}$

L'Hospital:

$$\frac{(\sin x)'}{(x^2)'} = \frac{\cos x}{2x} \to \frac{1}{0^+} = \infty$$

(A) $+\infty$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

15. Evaluate the following limit:

$$\lim_{\boxed{x\to \pi/2}} \frac{1-\sin x}{\cos x}.$$

$x \to \frac{\pi}{2}$: $1-\sin x \to 0$, $\cos x \to 0$: indeterminate form $\frac{0}{0}$

L'Hospital:

$$\frac{(1-\sin x)'}{(\cos x)'} = \frac{-\cos x}{-\sin x} = \frac{\cos x}{\sin x} = 0$$

$x \to \frac{\pi}{2}$: $\cos x \to 0$, $\sin x \to 1$

(A) 0 (B) 1 (C) $+\infty$

(D) -1 (E) 1/2

16. Determine the number of inflection points of the graph of $y = x^2 - \dfrac{1}{x}$ on its domain.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$f(x) = x^2 - \frac{1}{x}$ defined for $x \neq 0$

$f'(x) = 2x + \frac{1}{x^2}$

$f''(x) = 2 - \frac{2}{x^3}$

$f''(x) = 0$

$2 - \frac{2}{x^3} = 0$

$2 = \frac{2}{x^3}$

$x^3 = 1$

$x = 1$

$f''\left(\frac{1}{2}\right) = 2\left(1 - \frac{1}{\left(\frac{1}{2}\right)^3}\right) = -14 < 0$

$+$ $-$ $+$

0 $\frac{1}{2}$ 1 ∞

← the interesting values x for concavity, $f''(x)$

$\frac{1}{2}$ test value

only inflection

∞ test value

$\lim_{x\to\infty} f''(x) = 2 - \frac{2}{\infty} = 2 > 0$

concavity changes, but not in domain

17. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 (C) 20, 40

(D) 26, 27 (E) None of the above

$y+2x=80 \quad y=80-2x$

Want to maximize $P = xy = x(80-2x)$

$P(x) = x(80-2x)$

$P'(x) = 80-4x$

$P'(x)=0$, $80-4x=0$, $80=4x$, $x=20$, $y=80-2x$
$=80-2\cdot 20$
$=40$

- Graph of $P(x)$ is upside-down parabola, so the critical number $x=20$ with $P'(20)=0$ is the tip, so absolute max.
- Standard method: Practical restrictions $x \geq 0$, $y \geq 0$, $x \leq 40$
  (y is $80-2x$)

Interval of interest: x in $[0, 40]$

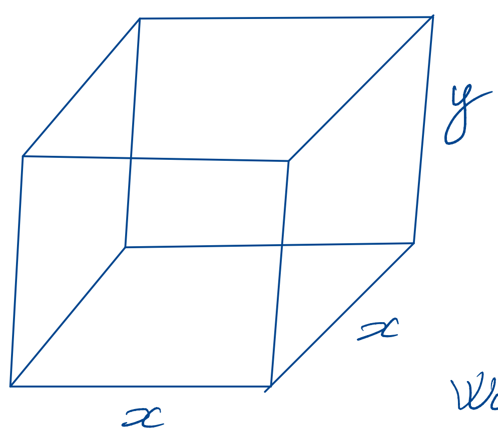
Interesting points

x	$P(x)$
0	0
20	$20\cdot 40 = 800$ ← clearly largest.
40	0

18. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?

(A) \$500 (B) \$600 (C) \$1000

(D) \$1200 (E) \$2000



$4000 = \text{Vol} = x^2 \cdot y \qquad y = \dfrac{4000}{x^2}$

$\text{Cost} = (\text{base} + 4 \text{ sides}) \cdot 1 = x^2 + 4xy$

no top

$C(x) = x^2 + 4xy = x^2 + 4x \cdot \dfrac{4000}{x^2}$

$= x^2 + \dfrac{16000}{x}$

$x \geq 0$ since it's a length.

Want min of $C(x)$ on $(0, \infty)$

y DNE for $x=0$

$C'(x) = 2x - \dfrac{16000}{x^2}$, $C'(x)=0$, $2x = \dfrac{16000}{x^2}$, $x^3 = 8000$,

$x=20$, $y = \dfrac{4000}{x^2} = \dfrac{4000}{20^2} = \dfrac{4000}{400} = 10$

x	$C(x)$
0	$\lim_{x\to 0^+} x^2 + \frac{16000}{x^2} = \infty$
20	$20^2 + \frac{16000}{20} = 400 + 800 = 1200$ smallest cost
∞	$\lim_{x\to\infty} x^2 + \frac{16000}{x} = \infty$

19. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x\to\infty} \frac{f(x)}{x^2} \to \infty$$

For L'Hospital need indeterminate form, so need $\lim_{x\to\infty} f(x) = \infty$ or $-\infty$

no limit   0   no limit ∞

(A) $\cancel{\sin(x)}$ (B) $\cancel{e^{-x}}$ (C) $\cancel{\cos(x)}$

(D) $\ln(x)$ (E) All of the above

20. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

(A) $1 - \ln 2$ (B) 0   (C) $2 - \ln 5$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

$v(t) = t - \ln(t^2+1)$

$v'(t) = 1 - \dfrac{2t}{t^2+1} = \dfrac{t^2+1-2t}{t^2+1}$

$v'(t) = 0$, $\quad t^2 - 2t + 1 = 0$, $\quad (t-1)^2 = 0$, $\quad t - 1 = 0$, $t = 1$

Points of interest on $[0, 2]$

t	$v(t)$
0	$0 - \ln 1 = 0$
1	$1 - \ln 2$
2	$2 - \ln 5$ largest

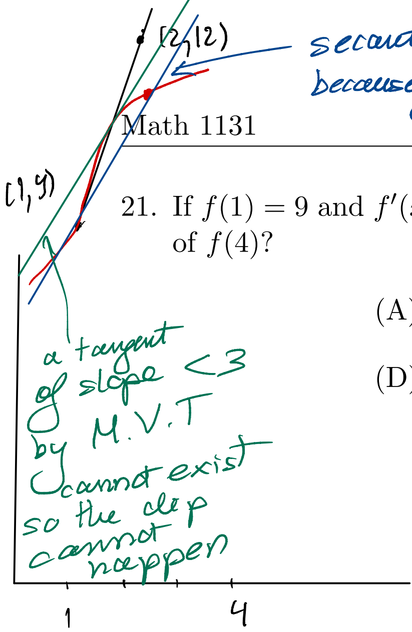
Which is bigger? $1 - \ln 2$ or $2 - \ln 5$

$1 - \ln 2 - (2 - \ln 5) = \ln \underbrace{\frac{5}{2}}_{2.5} - 1 < 0$, so $2 - \ln 5 > 1 - \ln 2 > 0$

$\underbrace{\log_e 2.5}$ $e \approx 2.7$, $\log_e 2.5 < 1$

$\log_e 2 < 1$

secant of slope < 3 because of dip below $y = 3x+6$ line



21. If $f(1) = 9$ and $f'(x) \geq 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

(A) 19 (B) 18 (C) 12

(D) Cannot be determined (E) None of the above

The graph of $y = f(x)$ cannot dip below line through $(1,9)$ of slope 3, equation $L(x) = 3x+6$

This is due to M.V.T.

So $f(4) \geq L(4) = 3 \cdot 4 + 6 = 18$

22. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0

critical numbers where $f'(x) = 0$

2nd deriv. test does not work. Use 1st? Not enough info

use 2nd deriv. test

local min local min local max

~~(A)~~ Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

~~(B)~~ Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

~~(D)~~ Local max at 4; local min at 0 No -3, -2,

~~(E)~~ Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6

has to be this by POE