Math 1131 Week 13 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

5.5: The Substitution Rule

1. Evaluate each of the following indefinite integrals using substitution, expressing your final answer in terms of x.

   (a) $\displaystyle\int x^2 \sin(x^3)\, dx$

   (b) $\displaystyle\int x\sqrt{4x+1}\, dx$

   (c) $\displaystyle\int \frac{x}{x^2+1}\, dx$

   (d) $\displaystyle\int \frac{1}{x \ln x}\, dx$

2. Rewrite each of the following definite integrals in x as a definite integral in the indicated new variable u. **Do not evaluate** the new definite integral.

   (a) $\int_0^1 x^2(1+2x^3)^5\,dx$ in terms of $u = 1 + 2x^3$

   (b) $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$ in terms of $u = \cos x$

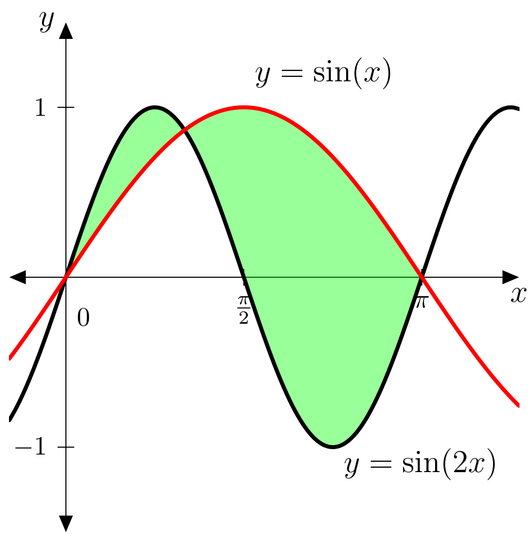
   (c) $\int_0^{\pi/3} \sin x \cos x\,dx$ in terms of $u = \cos x$

   (d) $\int_2^3 xe^{-x^2}\,dx$ in terms of $u = x^2$

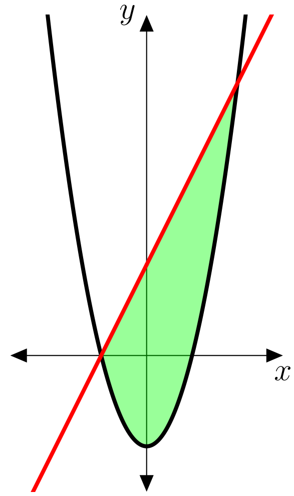
3. (T/F) When $u = \sqrt{x}$, $\int_0^4 f(\sqrt{x})\,dx = \int_0^2 2uf(u)\,du$.

6.1: Areas Between Curves

4. Find the area of the regions below, between $y = \sin x$ and $y = \sin(2x)$ for $0 \leq x \leq \pi$. (Hint: To find the exact coordinates of the point where the graphs cross, recall that $\sin(2x) = 2\sin x \cos x$.)



5. We want to find the area of the region bounded by $y = 2x + 4$ and $y = x^2 - 4$.



(a) Determine the coordinates of the points where the line and parabola intersect.

(b) Express the area as an integral with respect to x.

(c) Express the area as an integral with respect to y.

(d) Explain which of (b) or (c) is simpler to compute, and use the simpler one to find the area. Simplify your final answer.

Answers to Selected Problems:

1. (a) $-\frac{1}{3}\cos(x^3) + C$
   (b) $\frac{1}{40}(4x+1)^{5/2} - \frac{1}{24}(4x+1)^{3/2} + C$
   (c) $\frac{1}{2}\ln(x^2+1) + C$
   (d) $\ln(\ln x) + C$

2. (a) $\displaystyle\int_0^1 x^2(1+2x^3)^5\,dx = \int_1^3 \frac{1}{6}u^5\,du$
   (b) $\displaystyle\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx = \int_{1/2}^1 \frac{1}{u^2}\,du$
   (c) $\displaystyle\int_0^{\pi/3} \sin x\cos x\,dx = \int_{1/2}^1 u\,du$
   (d) $\displaystyle\int_2^3 xe^{-x^2}\,dx = \int_4^9 \frac{1}{2}e^{-u}\,du$

3. True

4. 5/2

5. (a) The points' x-coordinates are $x = -2$ and $x = 4$.
   (b) $\displaystyle\int_{-2}^4 ((2x+4) - (x^2-4))\,dx$
   (c) $\displaystyle\int_{-4}^0 (\sqrt{y+4} + \sqrt{y+4})\,dy + \int_0^{12} \left(\sqrt{y+4} - \frac{y-4}{2}\right)\,dy$
   (d) 36