Math 1131 Week 11 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

4.9: Antiderivatives

1. Find the most general antiderivative of the function (use C as any constant).

   (a) $f(x) = \dfrac{1}{2} + \dfrac{3}{4}x^2 - \dfrac{4}{5}x^3$

   (b) $f(x) = \dfrac{10}{x^9}$ for $x > 0$

   (c) $f(x) = \dfrac{x^4 + 3\sqrt{x}}{x^2}$ for $x > 0$

   (d) $f(x) = \cos x - 5\sin x + e^x$

   (e) $f(x) = e^2$

   (f) $f(x) = 7x^{2/5} + 8x^{-4/5}$ for $x > 0$

2. Find a function $f(x)$ satisfying the given conditions.

   (a) $f'''(x) = \cos x,\ f(0) = 1,\ f'(0) = 2$, and $f''(0) = 3$

   (b) $f''(x) = 2 - 12x,\ f(0) = 9,\ f(2) = 7$

3. A particle moves along a line according to the following information about its position $s(t)$, velocity $v(t)$, and acceleration $a(t)$. Find the particle's position function $s(t)$ for general t.

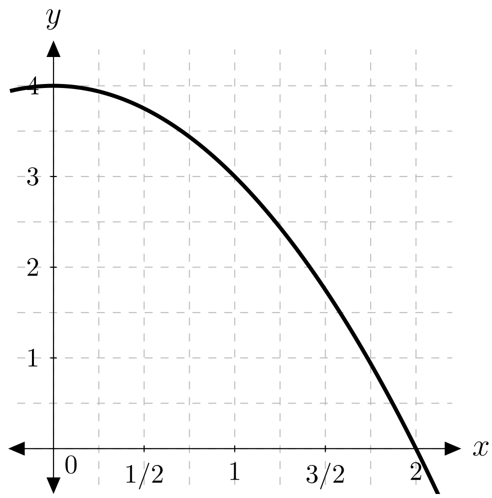
   (a) $v(t) = 1.5t^2 + 4t$, $s(4) = 50$

   (b) $a(t) = 3\cos t - 2\sin t$, $s(0) = 0$, $v(0) = 4$

4. T/F (with justification) The antiderivative of $\cos(x^2)$ is $\sin(x^2) + C$.

5.1: Areas and Distances

5. The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.



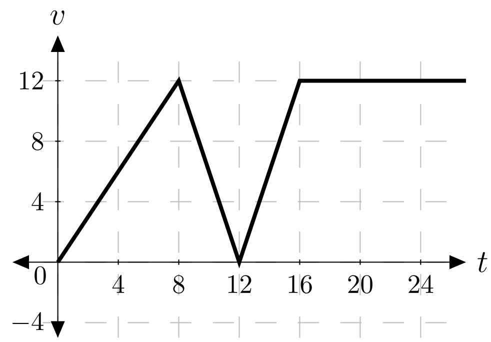
In (a), (b), and (c) below, estimate the area under the graph over $[0, 2]$ using 4 rectangles and the indicated type of endpoints: sketch the rectangles and then compute their areas. Determine from your sketch if the areas of the rectangles provide an underestimate or overestimate of the area under the curve, or if it can't be easily determined.

(a) Right endpoints.

(b) Left endpoints.

(c) Midpoints.

6. Here's a graph of the velocity (in ft/sec) of an object moving along a horizontal line.



(a) Over the interval $0 \leq t \leq 24$, determine the intervals when the object is speeding up and the intervals when the object is slowing down.

(b) From the end of Section 5.1 of the textbook, distance traveled is the area under the velocity vs. time graph. Use this to compute the distance traveled by the object from $t = 0$ to $t = 8$ seconds, in feet.

(c) Compute the distance traveled by the object from $t = 8$ to $t = 20$ seconds, in feet.

5.2: The Definite Integral

7. Evaluate the integral $\int_{1}^{4} (x+1)\, dx$ by drawing a picture and interpreting the integral in terms of areas.

8. Evaluate the integral $\int_{-3}^{3} (1+\sqrt{9-x^2})\, dx$ by drawing a picture and interpreting the integral in terms of areas.

9. T/F (with justification) A definite integral can be negative.

Answers to Selected Problems:

1. (a) $F(x) = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$
   (b) $F(x) = -\frac{5}{4}x^{-8} + C$
   (c) $F(x) = \frac{1}{3}x^3 - 6x^{-1/2} + C$
   (d) $F(x) = \sin x + 5\cos x + e^x + C$
   (e) $F(x) = e^2 x + C$
   (f) $F(x) = 5x^{7/5} + 40x^{1/5} + C$

2. (a) $f(x) = -\sin x + (3/2)x^2 + 3x + 1$
   (b) $f(x) = x^2 - 2x^3 + 5x + 9$

3. (a) $s(t) = \frac{1}{2}t^3 + 2t^2 - 14$
   (b) $s(t) = -3\cos t + 2\sin t + 2t + 3$

4. False

5. (a) 4.25; underestimate
   (b) 6.25; overestimate
   (c) 5.375; can't be easily determined to be an overestimate or underestimate

6. (a) For the first 8 seconds, the object is speeding up. Between $t = 8$ and $t = 12$, it's slowing down. From $t = 12$ to $t = 16$, it's speeding up.
   (b) 48 feet
   (c) 96 feet

7. 10.5

8. $6 + 4.5\pi$

9. True