University of Connecticut
Department of Mathematics

MATH 1131 PRACTICE PROBLEMS FOR EXAM 2 MULTIPLE CHOCE

Solutions

Sections Covered: 2.7, 2.8, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.8

Read This First!

- These practice problems are NOT sufficient review for the exam and do not represent the exact length of the exam. You should also use other resources, such as the textbook, worksheets, and Paul's Online notes to find further practice problems on topics that you have struggled with (or that you have trouble with on the practice problem set).
- Use these practice problems, in addition to other course materials, as a guide to determine what you need to study more deeply.
- The exam will contain some multiple choice questions as well as short-answer questions. **Short answer questions may be similar to questions found in lecture videos, live class activities, worksheets, and/or WebAsisgn.** When studying, make sure you are able to **fully justify your answers and reasoning** to prepare for the short-answer portion of the exam.
- The exam will be 50 minutes during your regular discussion section meeting.
- Please read each question carefully. For multiple choice questions, there is only one correct choice for each answer.
- On the exam, please carefully check all of your final answers. The submitted letter answers are the **ONLY** place that counts as your official answers for multiple-choice questions.
- You may **NOT** use a calculator or any other references on the exam, and **you are expected to work independently.**

1. If $f(x) = e^{4x}$, then evaluate the limit

$$\lim_{h\to 0} \frac{f(3+h) - f(3)}{h}. \rightarrow f'(3)$$

(A) e^{12} (B) e^{7} (C) $4e^{12}$

(D) $4e^{7}$ (E) $+\infty$

$f'(x) = 4e^{4x}$ (by chain Rule)

$f'(3) = 4 \cdot e^{12}$ C

2. If $f(x) = 3x^{10}$, then $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$ is which of the following? $\rightarrow f'(1)$

(A) $f'(x)$ (B) $f'(1)$ (C) Does not exist

(D) 0 (E) None of the above

B

3. If we want to calculate the derivative $f'(x)$ of $f(x) = 3x + 4$ using the limit definition of the derivative which of the following limits do we need to evaluate and to what does the limit evaluate?

(A) $\lim_{h\to 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 3$

(B) $\lim_{h\to 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 0$

(C) $\lim_{h\to 0} \frac{3h + 4 - (3x+4)}{h} = 3x + 3$

(D) $\lim_{h\to 0} \frac{3(x+h) + 4 - (3h+4)}{h} = 3$

(E) None of the above.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+4-(3x+4)}{h} = \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}} = 3$$

A

4. Below is the graph of the derivative $g'(x)$ of a function $g(x)$.

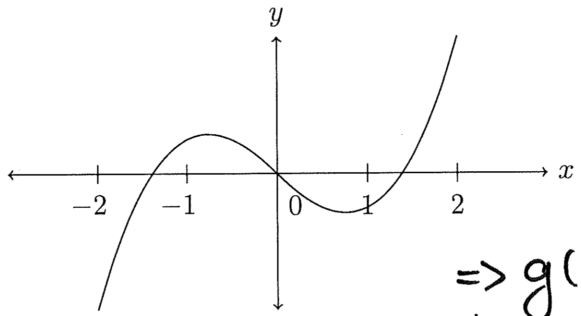


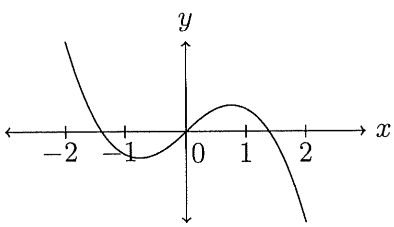
Figure 1: Graph of $g'(x)$.

$g'(x) = 0$ at $x \approx -1.4$ and $x \approx 1.4$

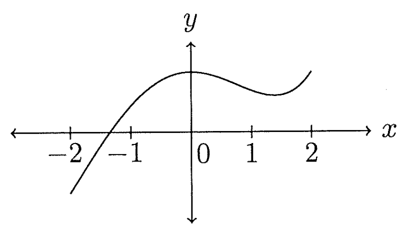
$\Rightarrow g(x)$ will have horizontal tangent lines at $x \approx -1.4$, $x \approx 1.4$

Which of the following is a possible graph of $g(x)$?

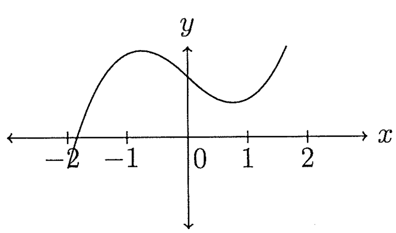
(A)



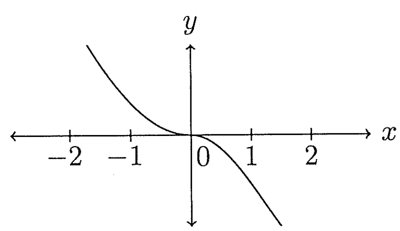
(B)



(C)



(D)



(E) None of the above

5. If $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ for $x > 0$, then $f'(4)$ is which of the following?

(A) $\frac{5}{4}$ (B) $\frac{3}{4}$ (C) $\frac{3}{16}$

(D) $\frac{255}{32}$ (E) $\frac{257}{32}$

$$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$\frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$

$$f'(4) = \frac{1}{2\cdot 2} - \frac{1}{2\cdot 4\cdot 2} = \frac{3}{16}$$

C

6. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

(A) 3 (B) 0 (C) 4

(D) 2 (E) 5

$$f'(x) = (3x^2 - 2x)(x^4 - x + 2) + (x^3 - x^2 + 1)(4x^3 - 1) \text{ (by Product Rule)}$$

$$f'(1) = (3 - 2)(2) + (1)(4 - 1) = 5$$

E

7. Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

(A) $y = \frac{1}{2}$ (B) $y = -\frac{1}{2}x + 1$ (C) $y = \frac{1}{2}x$

(D) $y = -\frac{1}{4}x + \frac{3}{4}$ (E) $y = \frac{1}{4}x + \frac{1}{4}$

point $x=1,\ y=\frac{1}{1+1}=\frac{1}{2}$

slope $\rightarrow$ $y' = \dfrac{1\cdot(x+1) - x\cdot 1}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

! Quotient Rule

$y'\big|_{x=1} = \dfrac{1}{2^2} = \dfrac{1}{4}$

eq: $y - \frac{1}{2} = \frac{1}{4}(x-1)$

$y = \frac{1}{4}x + \frac{1}{4}$ E

8. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

(A) 1 (B) -1 (C) 0

(D) 1/2 (E) $\sqrt{2}/2$

$f' = \cos x$

$f'' = -\sin x$

$f''' = -\cos x$

$f^{IV} = \sin x$

$f^{(5)} = \cos x$

$\vdots$

$125 \div 4 = 31$ R $\underline{\underline{1}}$

$f^{(125)}(x) = \cos x$

for $x = \pi$

$f^{(125)}(\pi) = -1$

B

9. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $g(x)$?

(A) x (B) x^2 (C) $\sin x$

(D) $\sin^2 x$ (E) None of the above

$\sin^2 x$ or $(\sin x)^2$

inner function: $g(x) = \sin x$ (C)

outer function: $f(x) = x^2$

10. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\frac{dy}{dx}$ when $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

(A) 9 (B) 12 (C) 13

(D) 15 (E) 23

! Product Rule

$y'(2) = f'(2)\,g(2) + f(2)\,g'(2)$

$y'(2) = 4 \cdot 1 + 3 \cdot 3 = 13$ (C)

11. If $g(x) = \dfrac{ax+b}{cx+d}$, then $g'(1)$ is which of the following? Note: The numbers a, b, c, and d are constants.

(A) $\dfrac{a+b-c-d}{c+d}$ (B) $\dfrac{ad-bc}{(c+d)^2}$ (C) $\dfrac{a+b-c-d}{(c+d)^2}$

(D) $\dfrac{ad+bc}{c+d}$ (E) $\dfrac{ad+bc}{(c+d)^2}$

! Quotient Rule

$$g'(x) = \frac{a(cx+d)-(ax+b)\cdot c}{(cx+d)^2} = \frac{ad-bc}{(cx+d)^2}$$

$$g'(1) = \frac{ad-bc}{(c+d)^2}$$ B

12. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\dfrac{3\pi}{4}$ (B) $\dfrac{3\pi}{4} + \dfrac{1}{2}$ (C) $\dfrac{1}{2}$

(D) $\dfrac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

! $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$, Product Rule

$$f'(x) = 3x^2 \arctan(x) + x^3 \cdot \frac{1}{1+x^2}$$

$$f'(1) = 3 \cdot \arctan(1) + \frac{1}{2} = 3 \cdot \frac{\pi}{4} + \frac{1}{2}$$ B

13. Consider the functions $f(x) = \sin(x^2)$ and $g(x) = \sin^2(x)$. Which of the following is true?

(A) $f'(x) = \cos(x^2)$ (B) $g'(x) = -2\sin(x)\cos(x)$ (C) $f'(x) = g'(x)$

(D) $f'(\pi) = g'(\pi) = 0$ (E) $f'(0) = g'(0)$

$f'(x) = 2x\cos(x^2)$

$f'(\pi) = 2\pi\cos(\pi^2)$

$f'(0) = 0$

$g'(x) = 2\sin(x)\cos(x)$

$g'(\pi) = 0$

$g'(0) = 0$

E

14. Find an equation of the tangent line to the curve $(x^2 + y^2)^2 = 4x^2y$ at the point $(1, 1)$.

(A) $y = 1$ (B) $y = x$ (C) $y = 2x - 1$

(D) $y = -x + 2$ (E) $y = -2x + 3$

point $(1, 1)$

slope $y'(1)$ → Implicit differentiation

$\rightarrow 2(x^2+y^2)(2x+2yy') = 8xy + 4x^2y'$

for $x=1$, $y=1$ $\rightarrow 2 \cdot 2(2+2y') = 8 + 4y'$

$8y' + 8 = 8 + 4y' \rightarrow y'(1) = 0$

eq → $y - 1 = 0(x-1)$ $y = 1$ A

15. Find $\frac{d}{dx}\left[\sin(\ln x^2)\right]$.

(A) $\frac{-\cos(\ln(x))}{x^2}$ (B) $\frac{-2\sin(\ln(x^2))}{x^2}$ (C) $\frac{\cos(\ln(x))}{2x^2}$

(D) $\frac{2\cos(\ln(x^2))}{x}$ (E) None of the above

! $(\ln u)' = \frac{1}{u}\cdot u'$ (or $\frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)}\cdot f'(x)$)

u = fct of x

$\to \cos(\ln x^2)\cdot[\ln x^2]' = \cos(\ln x^2)\cdot\frac{1}{x^2}\cdot 2x$ (D)

16. Find $\frac{d}{dx}\left[\log_4(3x)\right]$.

(A) $\frac{1}{3x\ln 4}$ (B) $\frac{1}{x\ln 4}$ (C) $\frac{1}{x}$

(D) $\frac{3}{x\ln 4}$ (E) $\frac{3}{x}$

! $\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}$

$\to \frac{d}{dx}\left[\log_4(3x)\right] = \frac{1}{(3x)\ln 4}\cdot(3x)' = \frac{1}{x\ln 4}$ (B)

17. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

I. $k > 0$

II. $P'(5) < 0$

III. $P'(10) = 100ke^{10k}$

$P'(t) = 100ke^{kt}$

(A) I and III only. (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

$P(5) > P(0) \rightarrow k > 0$ i ✓

$P'(5) = 100ke^{5k} > 0$ ii ✗

$P'(10) = 100 \cdot k \cdot e^{10k}$ iii ✓

(A)

18. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

(D) $10e^{-t\ln(2)/20}$ (E) $10e^{t\ln(2)/20}$

$A(t) = A_0 e^{kt}$

$\frac{1}{2} = e^{20k} \rightarrow k = -\frac{\ln 2}{20}$ $A(t) = 10e^{-t\frac{\ln 2}{20}}$ (D)

$A_0 = 10$

$A(20) = 5$

19. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by,

$k<0$

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ (E) $1013e^{h\ln(0.88)/1000}$

sea level $h=0$
$p(0)=1013$

$p(h)=p(0)e^{k\cdot h}$

$p(1000)=891.44=1013\,e^{1000k}$

$\rightarrow 0.88=e^{1000k} \rightarrow k=\frac{\ln(0.88)}{1000}$ $(k<0)$

$h=1000$
$p(1000)=88\%\,p(0)$
$=891.44$

$\Rightarrow p(h)=1013\,e^{h\cdot\frac{\ln(0.88)}{1000}}$

E

20. Determine $f''(x)$ for the function $f(x)=\frac{\ln x}{x^2}$. $\rightarrow$ Quotient Rule or Chain Rule + Product Rule $f(x)=x^{-2}\cdot\ln x$

(A) $\frac{-1}{2x^2}$ (B) $\frac{6\ln x}{x^4}$ (C) $\frac{1-6\ln x}{x^4}$

(D) $\frac{1-2\ln x}{x^3}$ (E) None of the above

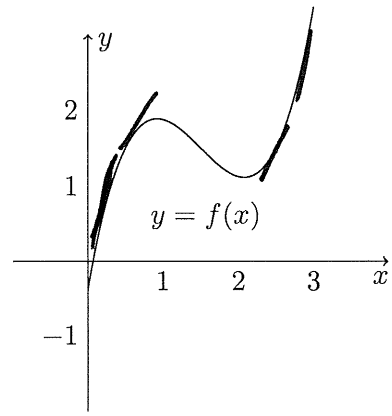
$f'(x)=\frac{\frac{1}{\cancel{x}}\cdot x^{\cancel{2}}-2x\ln x}{x^4}=\frac{1-2\ln x}{x^3}$

$f''(x)=\frac{-\frac{2}{\cancel{x}}\cdot x^{\cancel{3}2}+2\ln x\cdot 3x^2-3x^2}{x^6}=\frac{-5x^2+6x^2\ln x}{x^6}$

$=\frac{-5+6\ln x}{x^4}$

E

21. The curve below is the graph of $y = f(x)$. List all x-values, in interval form, on which $f'(x)$ (the *derivative* of f) is positive.



(A) $(0, 1)$ (B) $(0, 2)$ (C) $(1, 2)$

(D) $(2, 3)$ (E) $(0, 1)$ and $(2, 3)$

$f'(x)$ positive -> the tangent lines to the graph of $f(x)$ have positive slopes

-> when x in $(0, 1)$ and in $(2, 3)$ E