Math 1131 Week 9 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

4.3: How Derivatives Affect the Shape of a Graph

1. For the following functions, (i) determine all open intervals where $f(x)$ is increasing, decreasing, concave up, and concave down, and (ii) find all local maxima, local minima, and inflection points. Give all answers **exactly**, not as numerical approximations.

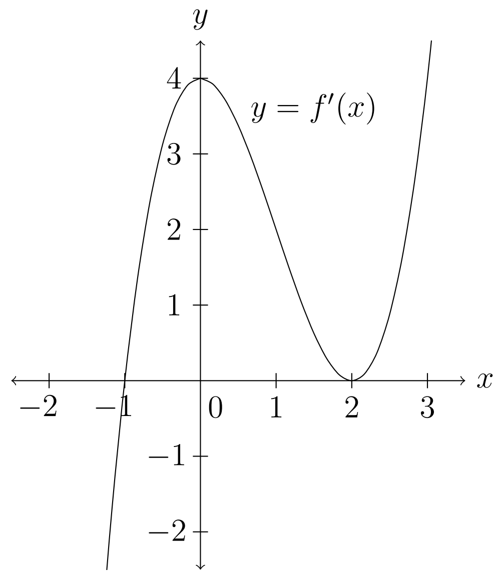
   (a) $f(x) = x^5 - 2x^3$ for all x

   (b) $f(x) = x - 2\sin x$ for $0 < x < 2\pi$

(c) $f(x) = e^{-x} - e^{-3x}$ for $x > 0$

2. For x in the interval $(0, 100)$, let $f(x) = x^{100} + (100 - x)^{100}$. Determine on what open intervals in $(0, 100)$ the function $f(x)$ is increasing and decreasing, and use this information to decide which of $33^{100} + 67^{100}$ or $41^{100} + 59^{100}$ is larger.

3. Below is a graph of $y = f'(x)$ for some function $f(x)$. Determine the intervals where $f(x)$ is increasing and decreasing, the x-values where $f(x)$ has local maxima and minima, and the x-values where $f(x)$ has inflection points.



4. T/F (with justification) If a function $f(x)$ on the interval $(-1, 1)$ is twice differentiable and $f''(c) = 0$ for some c in $(-1, 1)$ then $f(x)$ has an inflection point at $x = c$.

4.4: Indeterminate Forms and l'Hospital's Rule

5. For each of the following limits, indicate what kind of indeterminate form it is and then evaluate it with l'Hospital's rule.

   (a) $\lim_{x \to 0} \frac{(x+1)^{11} - 11x - 1}{x^2}$

   (b) $\lim_{x \to 0} \frac{\sin(3x)}{e^{9x} - e^{2x}}$

   (c) $\lim_{x \to 0} \frac{x - \tan x}{x - \sin x}$

(d) $\lim_{x\to\infty} \frac{\ln(1881x^2+1)}{\ln x}$

(e) $\lim_{x\to 0} \frac{\ln(\cos(2x))}{\ln(\cos(3x))}$

6. Indicate what kind of indeterminate form $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$ is and then try to evaluate it with l'Hospital's rule. Explain what goes wrong and then evaluate this limit using methods from earlier in the course.

Answers to Selected Problems:

1. (a) $f(x)$ increases on $(-\infty, -\sqrt{6/5})$ and $(\sqrt{6/5}, \infty)$ and decreases on $(-\sqrt{6/5}, \sqrt{6/5})$. It is concave up on $(-\sqrt{3/5}, 0)$ and $(\sqrt{3/5}, \infty)$, and concave down on $(-\infty, -\sqrt{3/5})$ and $(0, \sqrt{3/5})$.

   The function $f(x)$ has a local maximum at $-\sqrt{6/5}$ and a local minimum at $\sqrt{6/5}$, and it has inflection points at $x = -\sqrt{3/5}, 0, \sqrt{3/5}$.

   (b) $f(x)$ is increasing on $(\pi/3, 5\pi/3)$ and decreasing on $(0, \pi/3)$ and $(5\pi/3, 2\pi)$. It has a local maximum at $5\pi/3$ (with value $5\pi/3 + \sqrt{3} \approx 6.96$) and a local minimum at $\pi/3$ (with value $\pi/3 - \sqrt{3} \approx -.684$).

   $f(x)$ is concave up on $(0, \pi)$ and concave down on $(\pi, 2\pi)$; it has an inflection point at $x = \pi$.

   (c) $f(x)$ is increasing on $(0, (\ln 3)/2)$ and decreasing on $((\ln 3)/2, \infty)$, and it is concave down on $(0, \ln 3)$ and concave up on $(\ln 3, \infty)$. It has a local maximum at $(\ln 3)/2$, no local minimum, and an inflection point at $x = \ln 3$.

2. $f(x)$ is decreasing on $(0, 50)$ and increasing on $(50, 100)$. $33^{100} + 67^{100}$ is larger.

3. $f(x)$ is increasing for $x > -1$ and decreasing for $x < -1$. It has a local minimum at $x = -1$ and *no* local maximum. $f(x)$ has inflection points at $x = 0$ and $x = 2$.

4. False

5. (a) It is $0/0$. The limit is 55.

   (b) It is $0/0$. The limit is $3/7$.

   (c) It is $0/0$. The limit is -2.

   (d) It is ∞/∞. The limit is 2.

   (e) It is $0/0$. The limit is $4/9$.

6. It is ∞/∞. The limit is 1.