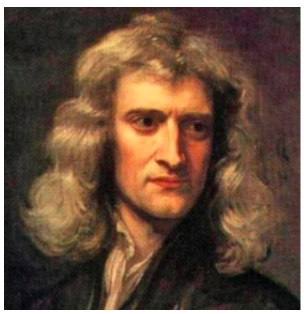
Math 1131: Calculus Overview

Calculus was created by Isaac Newton and Gottfried Leibniz in the 1600s. It has two parts, differential and integral calculus, which together may be called infinitesimal calculus (or "*the* Calculus" if you want to sound pretentious). The word "calculus" really means "system of calculation" (from the Latin word for "small stone") and there are many types of calculus in math: variational calculus, functional calculus, propositional calculus, and so on. The particular calculus of Newton and Leibniz was so influential that the undecorated term "calculus" refers to that.

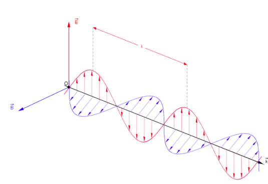


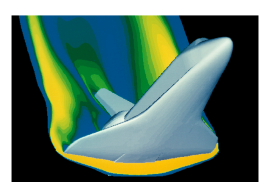
Newton

Leibniz

Newton and Leibniz worked on calculus largely independently (with different motivations, in physics and in geometry), but how much each knew about what the other was doing became a long-running controversy. Some aspects of calculus were known before Newton and Leibniz (ideas in integral calculus can be traced back to ancient Greece), but we give credit to Newton and Leibniz because they were the first to realize the generality of calculus to solve problems.

What makes calculus different from earlier topics in math is its relentless use of *infinity*: the infinitely small and the infinitely large. Calculus lets us break problems into infinitely many small parts, solve those, and then put them back together. What we can do with calculus that we can't do with algebra alone is model *dynamic* change. Without calculus, we can study cars at a *constant* velocity, circuits with a *constant* current, or *constant* fluid flow (the fluid could be a liquid or gas, like air). With calculus we can study *varying* velocity, *varying* current, and *varying* fluid flow.





A TED talk by Jeff Heys here gives examples of real world mathematical models that use calculus. Non-invasive medical procedures such as MRI depend on calculus (and physics and engineering). In a book listing what people thought was the greatest invention in the last 2000 years, there are many answers (Internet, universal education, *etc.*) and Bart Kosko's reply is calculus. He wrote "The world today would be very different if the Greeks and not Newton/Leibniz had invented or discovered calculus. The world today might have occurred a millennium or two earlier."

While calculus is the math of dynamic change, it helps solve problems that at first may not appear to be about anything dynamic. An example is optimization problems: find the input that gives a function its largest or smallest value. The way calculus is used in such a problem is by *thinking* about the problem in a dynamic way, using calculus to solve the dynamic problem, and then finishing the solution with some algebra. Optimization problems solved by calculus arise in economics (constrained optimization), machine learning (gradient descent), and statistics (least squares).[1]

Remark. Thanks to fast computers, calculus-based formulas in solutions to problems are often hidden in computer code, so people who use calculus-based algorithms in software on a computer may have no idea how the algorithms work, or may even think no fancy math is involved. Someone who wants to improve the algorithm, however, would need to understand it. Here's an analogy: you can be good at driving a car without knowing how a car works, but this doesn't make you qualified to be an automotive engineer.

A few short essays on the most basic ideas of calculus written by applied math professor Steven Strogatz are here, here, and here, and he wrote a book about the societal impact of calculus (supporting and supported by developments in science and technology). A video series on the "essence of calculus" created by YouTube math educator Grant Sanderson, a.k.a. 3Blue1Brown, is here. Strogatz's first essay and Sanderson's first video present different ways to explain why the area of a circle is πr^2, which are well worth comparing. Some topics covered by Sanderson (*e.g.*, Taylor series) are taught in second-semester calculus, so if you look over all of his calculus videos now, plan to go back and watch them again as you learn more.

In every calculus course, some students do well and some students don't. There

[1]The mathematical description of many optimization applications needs multivariable calculus. That is beyond the scope of this course, which is only about single-variable calculus. Multivariable calculus requires a solid understanding of the calculus in this course first.

are a few reasons that students struggle in this course, mostly connected to how well prerequisite material has been mastered.

1. The main reason students struggle in calculus is not calculus itself, but *algebra and trigonometry*, which are used all the time when solving calculus problems. Not having a good command of algebra and trigonometry (related to the unit circle) makes learning calculus much harder.

2. The main objects in calculus are not numbers, but *functions*: polynomials, exponentials, logarithms, and so forth. Every calculus problem is about a function or uses a function in its solution. You need to be able to operate with functions in the same way you do with numbers, *e.g.*, rewriting $1/x + 1/(x-1)$ as a single ratio $f(x)/g(x)$ in the same way you rewrite $1/2 + 1/7$ as a single fraction (it's $9/14$ – can you do that without a calculator?). Understand function notation and know the graphs of basic functions so that function formulas are not just symbols. When you see $y = x$, $y = x^2$, $y = \sin x$, and $y = 2^x$, immediately have a picture of their graphs in your head.

3. To learn calculus, you *must* solve lots and lots of calculus problems. This can not be overemphasized. Solve enough problems to understand what concepts mean and how they work, and then solve some more problems just to be sure. Watching someone else solve a calculus problem or reading someone else's solution that you have not put effort into first is not enough. It may be a start, but it is not where the real learning takes place. Like all math classes, a calculus course is cumulative: a topic that you decide you don't have time to study in one week might be an essential tool to solve problems a few weeks later.

In summary, to do well in this course here are three pieces of advice:

- you need to be comfortable using algebra and trigonometry,
- you need to become as comfortable working with functions as with numbers,
- you need to solve many calculus problems to understand what things mean and then solve a few more problems.