1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

   (A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ $\boxed{\text{(B) } x_{n+1} = (x_n^2 + 7)/(2x_n)}$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

   (D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$$\left.\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\ f(x_n) &= x_n^2 - 7 \\ f'(x_n) &= 2x_n \end{aligned}\right\} \to x_{n+1} = \cancel{x_n} - \frac{x_n^2 - 7}{2x_n} = \frac{2x_n^2 - (x_n^2 - 7)}{2x_n} = \underline{\underline{\frac{x_n^2 + 7}{2x_n}}}$$

2. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

   (A) 1/2 $\boxed{\text{(B) } 19/6}$ (C) 15/4

   (D) 12/7 (E) 17/6

$f(x_1) = f(3) = 3^2 - 10 = -1$

$f'(x) = 2x \to f'(x_1) = f'(3) = 2 \cdot 3 = 6$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{(-1)}{6}$

$= \frac{18}{6} + \frac{1}{6} = \underline{\underline{\frac{19}{6}}}$

3. Which of the following is the absolute maximum value of the function $f(x) = \dfrac{x}{x^2 + 4}$ on the interval $[0, 4]$?

   (A) $\frac{1}{8}$ (B) $\frac{1}{5}$ $\boxed{\text{(C) } \frac{1}{4}}$

   (D) $\frac{1}{2}$ (E) 1

$f'(x) = \frac{1 \cdot (x^2+4) - x \cdot 2x}{(x^2+4)^2} = \frac{4 - x^2}{(x^2+4)^2}$

crit #s: $4 - x^2 = 0 \to x = \pm 2$
(denom. never zero)

only one crit. # in $[0,4]$: $x = 2$

so $f(0) = 0$

$f(2) = \frac{2}{4+4} = \boxed{\frac{1}{4}} \to$ (largest) abs. max

$f(4) = \frac{4}{16+4} = \frac{1}{5}$

4. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

(A) 9 (B) $\sqrt{27}$ $\boxed{\text{(C) } \sqrt{3}}$

(D) 3 (E) No such value of c exists.

1) f cont. on $[0,3]$ ✓
2) f diff'ble on $(0,3)$ ✓
So MVT applies ✓

$f'(c) = \dfrac{f(3) - f(0)}{3 - 0}$

$3c^2 = \dfrac{3^3 - 0^3}{3 - 0} \rightarrow 3c^2 = \dfrac{27}{3} = 9$

$\rightarrow c^2 = 3$

$c = \pm\sqrt{3}$

$c = -\sqrt{3}$ not in $[0,3]$, so $\underline{c = \sqrt{3}}$

5. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

$\boxed{\text{(A) } 1}$ (B) -2 (C) $-2,\ 1$

(D) $-2,\ \dfrac{1}{2}$ (E) $-2,\ \dfrac{1}{2},\ 1$

$f'(x) = 6x^2 + 6x - 12$
$= 6(x^2 + x - 2)$
$= 6(x+2)(x-1) = 0$ (~~DNE~~)
$x = -2, 1$

2nd deriv. test:

$f''(x) = 12x + 6$

$f''(-2) = -24 + 6 = -18$ loc. max at $x = -2$

$f''(1) = 12 + 6 = 18$ loc min at $\underline{x = 1}$

6. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1 $\boxed{\text{(C) } 2}$

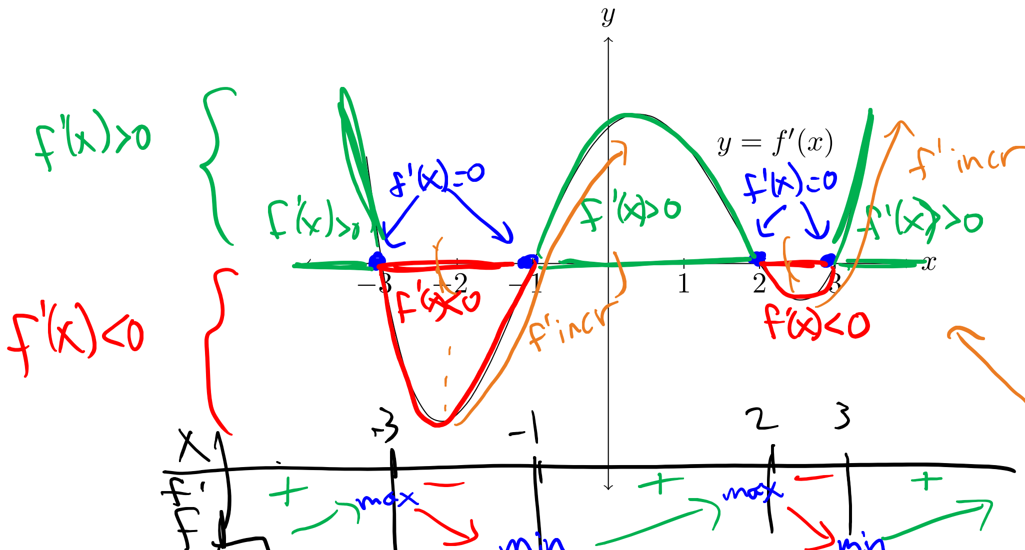
(D) 3 (E) 4

$f'(x) = 4x^3 - 16x$
$f''(x) = 12x^2 - 16 = 0$ (~~DNE~~)
$12x^2 = 16$
$x^2 = \frac{4}{3}$
$x = \pm\frac{2}{\sqrt{3}}$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	−	+
f	$\cup$	$\cap$	$\cup$

IP 1 IP 2

7. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



$\boxed{\text{(A)}}$ Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

8. Referring to the same graph of the derivative in question 7, at approximately what x-value(s) is $f(x)$ concave up? need $f''>0$, so f' increasing (i.e. $(f')'>0$).

2 intervals: $(-2.1, 0.8)$ and $(2.6, \infty)$

(A) $x < -1$ and $x > 1.5$

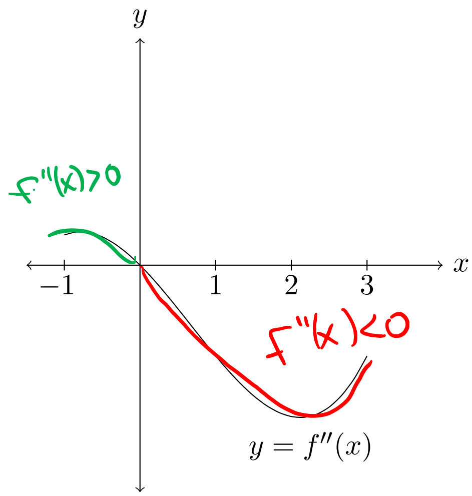
(B) $-1 < x < 2$

$\boxed{\text{(C)}}$ $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

9. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. f''>0 here ✓

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. f''<0 here ✓

(C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to − here ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to − here (i.e., f' changes ↗ to ↘ here) ✓

$\boxed{\text{(E)}}$ All of the above.

10. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only $\boxed{\text{(B) } (1, 2) \text{ only}}$ (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$ (DNE)

$x = 1, 2$

x	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
f''	+	−	+
f	∪	∩	∪
		conc. down	

11. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}. = \text{"}\frac{0}{0}\text{"}$$ Use L'Hospital's rule.

$\boxed{\text{(A) } +\infty}$ (B) $-\infty$ (C) 0
(D) 1/2 (E) $-1/2$

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cos x}{2x}$$

$$= \frac{\text{about } 1}{\text{small } + \to 0} = \underline{+\infty}$$

12. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}. = \text{"}\frac{0}{0}\text{"} \to \text{use L'Hospital's}$$

$\boxed{\text{(A) } 0}$ (B) 1 (C) $+\infty$
(D) -1 (E) 1/2

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x}$$

$$= \frac{-0}{-1} = \underline{0}$$

13. Determine the number of inflection points of the graph of $y = x^2 - \dfrac{1}{x}$ on its domain.

(A) 0 $\boxed{\text{(B) } 1}$ (C) 2 (D) 3 (E) 4

$$y' = 2x + \frac{1}{x^2} \to y'' = 2 - \frac{2}{x^3} = 0 \text{ or undefined}$$

$y'' = 0$:
$2 = \frac{2}{x^3}$
$x^3 = 1$
$x = 1$

y'' undef:
$x = 0$

x	$(-\infty, 0)$	asymptote	$(0, 1)$	IP	$(1, \infty)$
y''	$+$		$-$		$+$
y	$\cup$		$\cap$		$\cup$

14. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 $\boxed{\text{(C) 20, 40}}$

(D) 26, 27 (E) None of the above

maximize

$P = xy$ with $y+2x=80 \rightarrow y = 80-2x$

$P(x) = x(80-2x) = 80x - 2x^2 \rightarrow$ maximize on $(0,40)$

$P'(x) = 80 - 4x = 0$ (or ~~DNE~~)

$80 = 4x$

$\underline{x=20} \rightarrow y = 80 - 2(20) = \underline{40}$

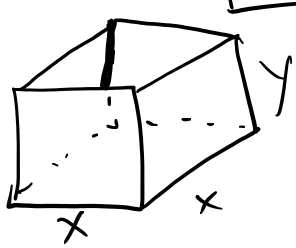
2nd deriv. test:

$P''(x) = -4 < 0$

15. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?

(A) \$500 (B) \$600 (C) \$1000

$\boxed{\text{(D) \$1200}}$ (E) \$2000



$V = x^2 y = 4000 \rightarrow y = \frac{4000}{x^2}$

Cost $= (\$1)$(total area of bottom + sides)

$= x^2 + 4xy$

$\rightarrow C(x) = x^2 + 4x\left(\frac{4000}{x^2}\right) = x^2 + \frac{16000}{x}$: minimize over $(0,\infty)$

$C'(x) = 2x - \frac{16000}{x^2} = 0$ or ~~DNE~~ ~~$x=0$ (not in domain)~~

$\rightarrow 2x = \frac{16000}{x^2}$

$x^3 = 8000$

$\underline{x = 20} \rightarrow$ min. cost $= C(20) = 20^2 + \frac{16000}{20}$

$= 400 + 800 = \underline{1200}$

$C''(x) = 2 + \frac{32000}{x^3} \rightarrow C''(20) > 0 \rightarrow$ minimum ✓

16. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x\to\infty} \frac{f(x)}{x^2} = \text{"}\frac{?}{\infty}\text{"} \to \text{L'H applies if } \lim_{x\to\infty} f(x) = \pm\infty$$

(A) $\sin(x)$ (B) e^{-x} (C) $\cos(x)$

$\boxed{\text{(D) } \ln(x)}$ (E) All of the above

~~A)~~ $\lim_{x\to\infty} \sin x$ DNE

~~B)~~ $\lim_{x\to\infty} e^{-x} = 0$

~~C)~~ $\lim_{x\to\infty} \cos x$ DNE

D) $\lim_{x\to\infty} \ln x = \infty$ ✓

17. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

(A) $1 - \ln 2$ (B) 0 $\boxed{\text{(C) } 2 - \ln 5}$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

$$v'(t) = 1 - \frac{2t}{t^2+1} = 0$$ (or DNE: never since $t^2+1>0$ always)

$$1 = \frac{2t}{t^2+1}$$

$$t^2+1 = 2t$$

$$t^2-2t+1=0$$

$$(t-1)^2=0$$

$$t=1$$

max out of $v(0), v(1), v(2)$:

$v(0) = 0 - \ln(1) = 0$

$v(1) = 1 - \ln(2) > 0$

$v(2) = 2 - \ln(5) > 0$

which is larger?

0 2

x			
v'	+	+	
v	↗	↗	

max @ $x = 2$ → $v(2) = 2 - \ln(5)$ is max value

18. If $f(1) = 9$ and $f'(x) \geq 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

(A) 19 (B) 18 (C) 12

(D) Cannot be determined (E) None of the above

Using mean value theorem: $f'(c) = \frac{f(4)-f(1)}{4-1}$ for some c in $[1,4]$, so:

$3f'(c) = f(4) - f(1) \rightarrow f(4) = f(1) + 3f'(c)$
$= 9 + 3f'(c) \geq 9 + 3(3) = 18$

19. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0
		min $f''>0$	CBD $f''=0$	min $f''>0$		max $f''<0$	

crit #s, where $f'=0$.
$x = -3, -2, 0, 4$.

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6

20. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) 0 (B) 1 (C) -2

(D) 2 (E) -1

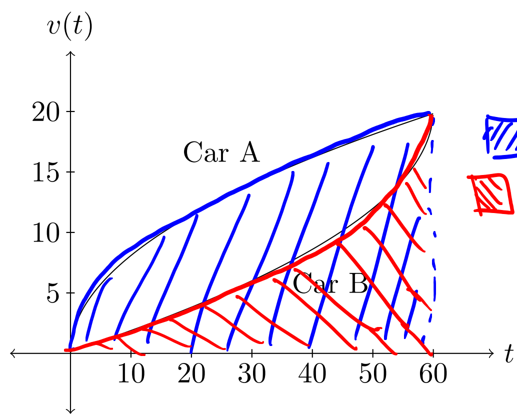
$f'(x) = 2x - \frac{3}{2}x^2 + C$, $f'(0) = C = -1 \to f'(x) = 2x - \frac{3}{2}x^2 - 1$

$f(x) = x^2 - \frac{1}{2}x^3 - x + D$, $f(0) = D = 1 \to f(x) = x^2 - \frac{1}{2}x^3 - x + 1$

$\to f(2) = 4 - \frac{1}{2}\cdot 8 - 2 + 1$

$= \underline{-1}$

21. Below is the graph of the velocity (measured in ft/sec) over the interval $0 \leq t \leq 60$ for two cars, Car A and Car B. How do the distances traveled by each compare over this interval?



area = A's distance travelled

area = B's distance travelled

(A) Car A travelled farther because its speed was increasing the whole time

(B) Car B travelled farther because its speed was increasing the whole time

(C) Car A travelled farther because the area under its velocity curve is larger than B's

(D) Car A and Car B travelled the same distance

(E) Car B travelled farther because it was moving faster at the end

22. Find $f(x)$ if $f'(x) = 3x^2 + \dfrac{2}{x}$ for $x > 0$ and $f(1) = 3$.

(A) $x^3 + 2\ln x$ (B) $x^3 - \dfrac{2}{x^2}$ (C) $x^3 - \dfrac{2}{x^2} + 4$

$\boxed{\text{(D) } x^3 + 2\ln x + 2}$ (E) $x^3 + 2\ln x + 3$

$f(x) = x^3 + 2\cdot\ln|x| + C$

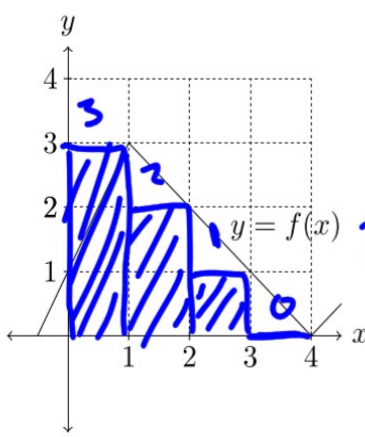
$\rightarrow f(x) = x^3 + 2\ln x + C \quad (x>0)$

$f(1) = 3 \rightarrow 1^2 + \underbrace{\cancel{2\ln 1}}_{0} + C = 3$

$1 + C = 3 \rightarrow C = 2$

$\rightarrow f(x) = \underline{x^3 + 2\ln x + 2}$

23. If we use a right endpoint approximation with four subintervals (i.e., R_4), then what is the resulting approximation for the area under the curve $y = f(x)$ from $x = 0$ to $x = 4$?



$3 + 2 + 1 + 0 = 6$

(A) 9 (B) 7 (C) 7.5

$\boxed{\text{(D) } 6}$ (E) 6.5