

Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

---

**Solutions should show all of your work, not just a single final answer.**

---

## 4.1: Maximum and Minimum Values

---

1. For the following functions, find all critical numbers **exactly**.

(a)  $f(x) = x^5 - 2x^3$

(b)  $f(x) = x - 2 \sin x$  for  $-2\pi < x < 2\pi$

(c)  $f(x) = e^{-x} - e^{-3x}$  for  $x > 0$

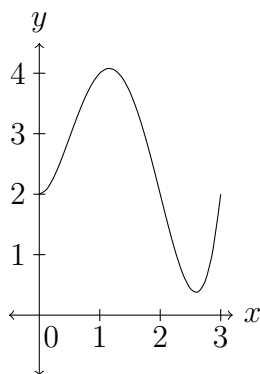
2. Use calculus to find the absolute maximum and minimum values of the following functions on the given intervals. Give your answers **exactly** and show supporting work.

(a)  $f(x) = x^3 - 2x^2 + x + 1$  on  $[0, 1]$

(b)  $f(x) = x^4 - 2x^2 + 4$  on  $[0, 2]$

(c)  $f(x) = (7x - 1)e^{-2x}$  on  $[0, 1]$

3. Below is the graph of  $f(x) = x^4 - 5x^3 + 6x^2 + 2$ . On the interval  $[0, 3]$  determine the maximum and minimum value of the *slope* of the graph, *i.e.*, the maximum and minimum values of  $g(x) = f'(x)$ .



4. T/F (with justification) If  $f(x)$  is a differentiable function on  $(a, b)$  and  $f(x)$  has a local maximum or minimum value at  $x = c$  in  $(a, b)$  then  $f'(c) = 0$ .
5. T/F (with justification) If  $f(x)$  is a differentiable function on  $(a, b)$  and  $f'(c) = 0$  for a number  $c$  in  $(a, b)$  then  $f(x)$  has a local maximum or minimum value at  $x = c$ .

---

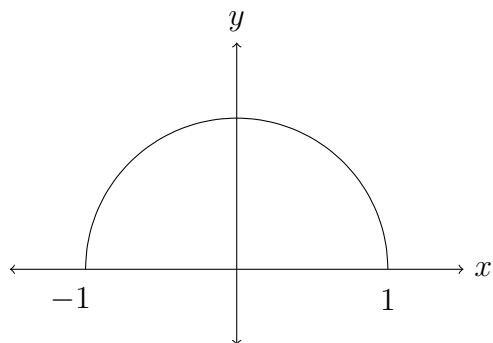
## 4.2: Mean Value Theorem

---

6. Find every number  $c$  that satisfies the conclusion of the Mean Value Theorem for the function  $f(x) = x^3 - 4x^2 - 5$  on the interval  $[1, 2]$ .

7. T/F (with justification) The function  $1 - \frac{1}{x^4}$  satisfies the hypotheses of Rolle's Theorem on the interval  $[-1, 1]$ .

8. T/F (with justification) The graph of the semicircle on  $[-1, 1]$  below fits the hypotheses of the Mean Value Theorem.



Answers to Selected Problems:

1. (a) 0 and  $\pm\sqrt{6/5}$   
(b)  $x = \pm\pi/3$  and  $x = \pm5\pi/3$   
(c)  $x = \frac{1}{2}\ln 3$
2. (a) On  $[0, 1]$  the absolute maximum value of  $f(x)$  is  $31/27$  and the absolute minimum value of  $f(x)$  is 1.  
(b) On  $[0, 2]$  the absolute maximum value of  $f(x)$  is 12 and the absolute minimum value of  $f(x)$  is 3.  
(c) The absolute maximum value of  $f(x)$  is  $(7/2)e^{-9/7}$ , and the absolute minimum value of  $f(x)$  is  $-1$ .
3.  $f'(x) = g(x)$  has maximum value 9 and minimum value  $-4$ .
4. True
5. False
6.  $c = 5/3$
7. False
8. True