Math 1131 Week 6 Worksheet

Name: _____

Discussion Section:_____

Solutions should show all of your work, not just a single final answer.

3.6: Derivatives of Logarithmic Functions

1. Compute dy/dx. Write your final answers entirely in terms of x.

   (a) $y = \ln(2 + \sin x)$

   (b) $y = \ln(\ln x)$

   (c) $y = 5^x$

2. T/F (with justification) If $f(x) = \ln(x^2)$ for all $x > 0$ then $f'(x) = \dfrac{1}{x^2}$.

3. T/F (with justification) If $f(x) = 10^x$ for all x then $f'(x) = x10^{x-1}$.

3.8: Exponential Growth and Decay

4. In 1859, 24 rabbits were released into the wild in Australia, where they had no natural predators. Their population grew exponentially, doubling every 6 months.

   (a) Determine $P(t)$, the function that gives the population at time t, **and** the differential equation describing the population growth. Let units for t be years since 1859.

   (b) After how many years, rounded to one digit after the decimal point, did the rabbit population reach 1,000,000?

   (c) Determine the *rate* of population change, in rabbits/year, midway through the third year. (**Warning**: t is not 3.5, just like the year midway through the 21st century is not 2150.) Round the final answer to 2 digits after the decimal point.

5. The element Unobtainium has a half-life of 3 years. Let $M(t)$ be the mass of Unobtainium at time t starting with an initial amount of 14 kg.

   (a) Give a formula for $M(t)$.

   (b) After how many years will the initial mass of Unobtainium shrink to 1 kg? Round your answer to one digit after the decimal point.

6. Starbucks serves coffee at 170° and the room temperature in Starbucks is 70°. The coffee cools to 100° after 10 minutes. Let $T(t)$ be the temperature of the coffee at time t, measured in minutes.

   (a) Write down the differential equation for $T(t)$ and determine a formula for $T(t)$.

   (b) From the time when the temperature is 100° at $t = 10$, how many *additional* minutes will it take for the temperature of the coffee to reach 80°? Round your answer to one digit after the decimal point.

7. T/F (with justification) If $\dfrac{dy}{dx} = y$ then $y = 0$ or $y = e^x$.

8. T/F (with justification) A function $y(t)$ satisfying $\dfrac{dy}{dt} = -.01y$ has $\lim\limits_{t\to\infty} y(t) = 0$.

Answers to Selected Problems:

1. (a) $\frac{\cos x}{2+\sin x}$
   (b) $\frac{1}{x \ln x}$
   (c) $(\ln 5)5^x$

2. False

3. False

4. (a) $P(t) = 24e^{(2 \ln 2)t}, \quad \frac{dP}{dt} = (2 \ln 2)P$
   (b) ≈ 7.7 years
   (c) $\approx 1064.7 \frac{\text{rabbits}}{\text{year}}$

5. (a) $M(t) = 14e^{-((\ln 2)/3)t}$
   (b) ≈ 11.4 (years)

6. (a) $\dfrac{dT}{dt} = \dfrac{\ln(.3)}{10}(T - 70)$ and $T(t) = 70 + 100e^{(\ln(.3)/10)t}$.
   (b) It takes about 9.1 additional minutes for the coffee's temperature to reach 80°.

7. False

8. True