Math 1131 Week 5 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

3.3: Derivatives of Trigonometric Functions

1. Compute the derivative of each function below using differentiation rules.

   (a) $f(x) = x^3 \cos x$

   (b) $f(x) = \dfrac{1 + \sin x}{1 + \cos x}$

   (c) $f(x) = e^x \tan x$

   (d) $f(x) = \dfrac{\sec x}{\sqrt{x}}$ (Compute (d) in **two ways**, using (i) the quotient rule and (ii) the product rule.)

2. Find the equation of the tangent line to the curve $y = \sin x \cos x$ at $x = \dfrac{\pi}{4}$. (Your coefficients must be exact, not approximations.)

3. Find the higher derivative $\dfrac{d^{1881}}{dx^{1881}}(2\cos x)$ by finding the first eight derivatives and observing the pattern that occurs.

4. Determine the following limits by making a change of variables to allow you to use the relation $\lim\limits_{t\to 0}\dfrac{\sin t}{t} = 1$.

   (a) $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$

   (b) $\lim\limits_{x\to 0}\dfrac{\sin 7x}{5x}$

3.4: The Chain Rule

5. Compute the derivative with respect to x of each function below using differentiation rules.

   (a) $f(x) = (x^3 - x + 1)^{10}$

   (b) $f(x) = \sqrt{x^3 + 4x}$

   (c) $f(x) = e^{ax}\cos(bx)$ for constants a and b

   (d) $f(x) = \left(\dfrac{e^x}{3 - x}\right)^8$

   (e) $f(x) = \sin^2(x) - \sin(x^2)$

6. Differentiate the functions below **with respect to** t, where $r = r(t)$ is a function of t.

   (a) $(r^2 + 1)^4$

   (b) $\sin(2r) - 2\sin r$

   (c) e^{r^2+ar+b} for constants a and b.

7. If $f'(0) = 5$ and $F(x) = f(3x)$, what is $F'(0)$?

8. T/F (with justification) If $f(x)$ is differentiable, then $\dfrac{d}{dx}(f(\sqrt{x})) = \dfrac{f'(x)}{2\sqrt{x}}$.

3.5: Implicit Differentiation

9. Find $\dfrac{dy}{dx}$ using implicit differentiation. Your final answer may involve both x and y.

   (a) $x^2y - axy^2 = x + y$ where a is a constant.

   (b) $\sin(x + y) = x + \cos(3y)$

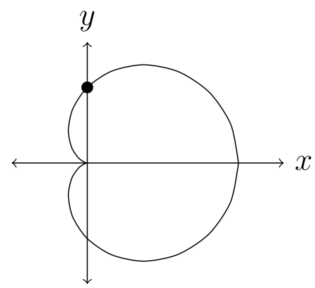
   (c) $e^{xy} = x^2 + y^2$

   (d) $x = \arctan(y^2)$

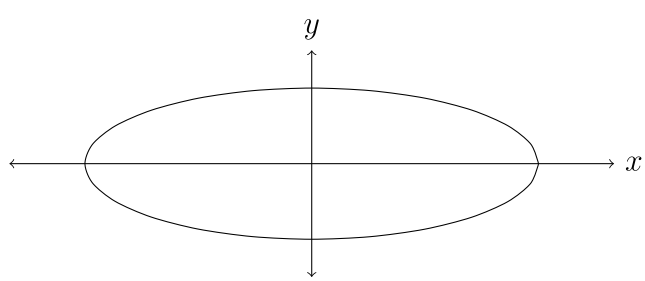
10. Use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

at the point $(0, 1/2)$. **Note**. The graph of this equation is known as a cardioid, shown below. It's not the graph of a function, and this is where implicit differentiation can be helpful to us.



11. On the ellipse $x^2 + 9y^2 = 9$, find $\dfrac{d^2y}{dx^2}$ using implicit differentiation. Your final answer may involve both x and y.



Answers to selected problems

1. (a) $(x^3 \cos x)' = (x^3)' \cos x + x^3(\cos x)' = 3x^2 \cos x - x^3 \sin x$
   (b) $\frac{1+\cos x+\sin x}{(1+\cos x)^2}$
   (c) $e^x(\tan x + \sec^2 x)$
   (d) $\frac{x \sec x \tan x - (\sec x)/2}{x\sqrt{x}}$

2. $y = \frac{1}{2}$

3. $-2 \sin x$.

4. (a) 4
   (b) $\frac{7}{5}$

5. (a) $10(x^3 - x + 1)^9(3x^2 - 1)$
   (b) $\frac{3x^2+4}{2\sqrt{x^3+4x}}$
   (c) $ae^{ax} \cos(bx) + e^{ax}(-b \sin(bx))$
   (d) $\frac{8e^{8x}(4-x)}{(3-x)^9}$
   (e) $2 \sin x \cos x - 2x \cos(x^2)$

6. (a) $8r(r^2 + 1)^3 \frac{dr}{dt}$
   (b) $2 \cos(2r) \frac{dr}{dt} - 2 \cos r \frac{dr}{dt}$
   (c) $e^{r^2+ar+b}(2r + a) \frac{dr}{dt}$

7. 15

8. False

9. (a) $\frac{dy}{dx} = \frac{1-2xy+ay^2}{x^2-2axy-1}$
   (b) $\frac{dy}{dx} = \frac{1-\cos(x+y)}{3\sin(3y)+\cos(x+y)}$
   (c) $\frac{dy}{dx} = \frac{2x-ye^{xy}}{xe^{xy}-2y}$
   (d) $\frac{y^4+1}{2y}$

10. $y = x + \frac{1}{2}$

11. $-\frac{1}{9y^3}$