

Name: _____

Discussion Section: _____

Solutions should show all of your work, not just a single final answer.

3.1: Derivatives of Polynomials and Exponential Functions

1. Use differentiation rules from Section 3.1 (**not other methods**) to compute the derivative of the following functions.

(a) $f(x) = 7x^3 - 5x + 8$

(b) $f(x) = e^x + x^e$

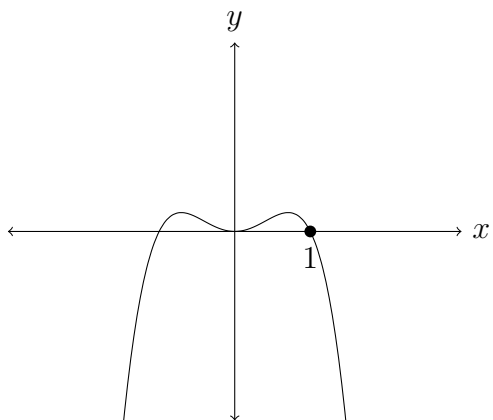
(c) $f(x) = 3x + \sqrt{3x}$

(d) $f(x) = \sqrt[4]{x} - 4e^x$

(e) $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

(f) $f(x) = \frac{12}{x^5} - \frac{7}{\sqrt[3]{x}}$

2. Use differentiation rules to find the equation of the tangent line to $y = x^2 - x^4$ (see below) at the point $(1, 0)$.



3. Use differentiation rules to find the values of a and b that make the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2, \\ ax^3 + bx & \text{if } x > 2 \end{cases}$$

differentiable at $x = 2$.

4. Find all points $(c, f(c))$ on the graph of $f(x) = x^3 - 3x^2$ where the tangent line has slope 9.

5. T/F (with justification) If $f(x) = \sqrt{7}$ for all x , then $f'(x) = \frac{1}{2\sqrt{7}}$ for all x .

3.2: The Product and Quotient Rules

6. Compute the derivative of each function below using the methods from Sections 3.1 and 3.2 (**not other methods**).

(a) $f(x) = \frac{x}{x+3}$ (simplify numerator in final answer)

(b) $f(x) = \frac{e^x}{1+e^x}$ (simplify numerator in final answer)

(c) $f(x) = \sqrt{x}e^x$

(d) $f(x) = \frac{e^x}{x^n}$ for constant n , in two ways: (i) quotient rule and (ii) product rule

(e) $f(x) = \frac{1}{x} + \frac{1}{1-x}$ (in final answer, use a common denominator and simplify numerator)

7. In the function $h(x)$ below, defined in terms of $f(x)$ and $g(x)$, determine $h'(2)$ in each case if $f(2) = 3$, $g(2) = 4$, $f'(2) = 1$, and $g'(2) = -5$.

(a) $h(x) = 2f(x) + 5g(x)$

(b) $h(x) = f(x)g(x)$

(c) $h(x) = \frac{f(x)}{g(x)}$

(d) $h(x) = \frac{g(x)}{f(x) + 2}$

Answers to selected problems

1. (a) $f'(x) = 21x^2 - 5$
(b) $f'(x) = e^x + ex^{e-1}$
(c) $f'(x) = 3 + \frac{\sqrt{3}}{2\sqrt{x}}$
(d) $f'(x) = \frac{1}{4x^{3/4}} - 4e^x$.
(e) $f'(x) = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x^{3/2}}$.
(f) $f'(x) = -\frac{60}{x^6} + \frac{7}{5x^{6/5}}$.
2. $y = -2x + 2$
3. $a = 1/4, b = 1$.
4. $(-1, f(-1)) = (-1, -4)$ and $(3, f(3)) = (3, 0)$.
5. False
6. (a) $\frac{3}{(x+3)^2}$
(b) $\frac{e^x}{(1+e^x)^2}$
(c) $\frac{(2x+1)e^x}{2\sqrt{x}}$
(d) (i): $\frac{x^n e^x - e^x n x^{n-1}}{x^{2n}}$
(ii): $e^x x^{-n} - n e^x x^{-n-1}$ (show why these are the same!)
(e) $\frac{2x-1}{x^2(1-x)^2}$
7. (a) -23
(b) -11
(c) $\frac{19}{16}$
(d) $-\frac{29}{25}$