Math 1131 Week 3 Worksheet

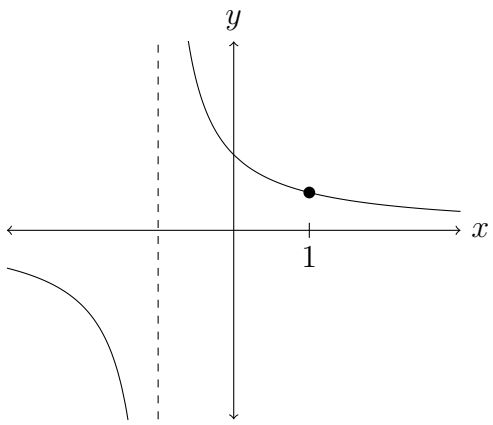
Name: _____

Discussion Section:_____

Solutions should show all of your work, not just a single final answer.

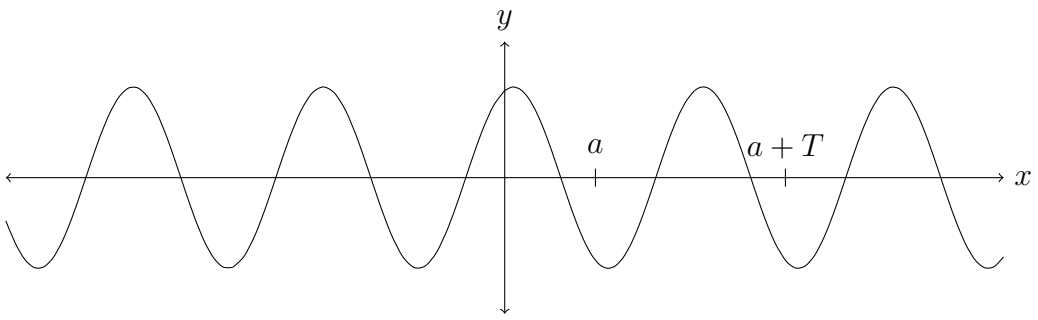
2.7: Derivatives and Rates of Change

1. The function $f(x) = \dfrac{1}{x+1}$ is graphed below. Find $f'(1)$ using the limit definition of the derivative (**no credit for using any other method**) and then find the equation of the tangent line to the graph at $x = 1$.



2. If a function $f(x)$ has $f(3) = 2$ and $f'(3) = 4$, work out an equation of the tangent line to the graph of $y = f(x)$ at the point $(3, f(3))$.

3. Below is the graph of a function $f(x)$ that's periodic: there's a $T > 0$, called the period, such that $f(x + T) = f(x)$ for all x. Explain why the derivative $f'(x)$ is also periodic with period T in two different ways.

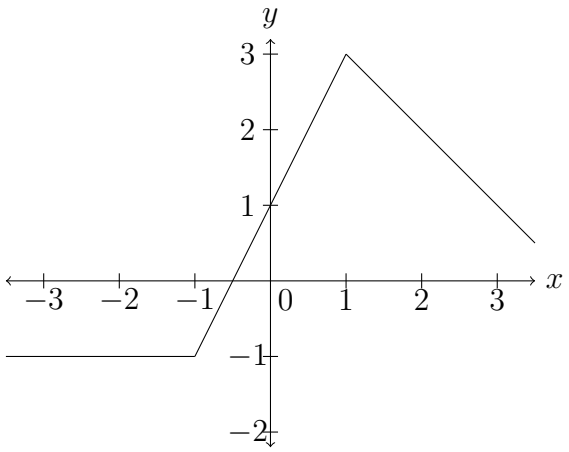


(a) using tangent lines to the graph of the function

(b) using the limit definition of the derivative

2.8: The Derivative as a Function

4. The graph of $y = f(x)$ is pictured below.



(a) Compute each derivative below, explaining your calculations. If a derivative does not exist, write DNE.

(a) $f'(-2)$

(b) $f'(1)$

(c) $f'(-1)$

(d) $f'(2)$

(e) $f'(0)$

(f) $f'(3)$

(g) Sketch a graph of the derivative $f'(x)$, leaving blank spots where it does not exist.

5. Find the derivative of $f(x) = \dfrac{8}{x^2}$ using the limit definition of the derivative (**no credit for using any other method**). Hint: Look at 3b on the Week 2 Worksheet.

6. T/F (with justification) A function that is continuous at a is also differentiable at a.

7. T/F (with justification) If $f'(2)$ exists, then $\lim_{x \to 2} f(x) = f(2)$.

Answers to Selected Worksheet Problems

1. $f'(1) = -\dfrac{1}{4}$, $y = -\frac{1}{4}x + \frac{3}{4}$.

2. $y = 4x - 10$.

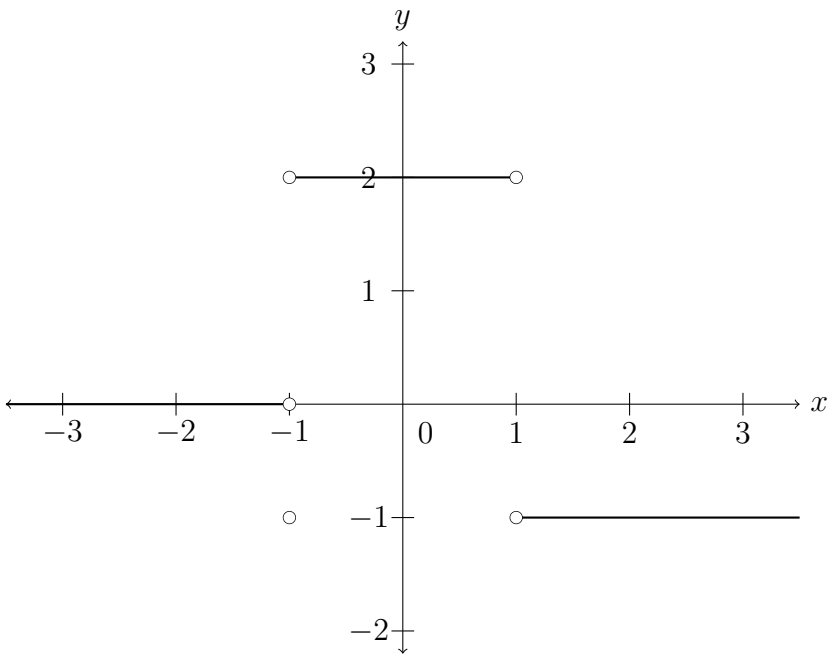
3. Contact us with your questions!

4. (a) (i): $f'(-2) = 0$.
   (ii): $f'(1)$ DNE.
   (iii): $f'(-1)$ DNE.
   (iv): $f'(2) = -1$.
   (v): $f'(0) = 2$
   (vi): $f'(3) = -1$.

   (b) Based on the work in (a),

$$f'(x) = \begin{cases} 0, \text{ if } x < -1, \\ 2, \text{ if } -1 < x < 1, \\ -1, \text{ if } x > 1. \end{cases}$$

   Below is the graph, with discontinuities at $x = \pm 1$.



5. $f'(x) = -16/x^3$.

6. False

7. True