Math 1131 Week 2 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

2.3: Calculating Limits Using the Limit Laws

1. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

(a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.

(b) Evaluate the following limits if they exist. (If a limit does not exist, write DNE.)

(i) $\lim_{x \to 1^-} f(x)$

(ii) $\lim_{x \to 1^+} f(x)$

(iii) $\lim_{x \to 1} f(x)$

(iv) $\lim_{x \to 2^-} f(x)$

(v) $\lim_{x \to 2^+} f(x)$

(vi) $\lim_{x \to 2} f(x)$

2. Evaluate the following limits exactly using algebra and limit laws (some limits may be DNE).

(a) $\lim_{x\to 2} \dfrac{x^3 - 2}{2x^2 - 3x + 2}$

(b) $\lim_{x\to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

(c) $\lim_{x\to 3} \dfrac{\sqrt{x^2 + 40} - 7}{x - 3}$

(d) $\lim_{x\to -2} \sqrt{x^4 + 3x + 6}$

(e) $\lim_{x\to 1} \dfrac{x^2 + 4x}{x^2 + 3x - 4}$

(f) $\lim_{x\to 1} \dfrac{(x^2 + x)^2 - 4}{x^2 + x - 2}$

3. Evaluate the following limits using algebra and limit laws (some limits may be DNE). Note that a represents a constant, and answers may be in terms of a.

   (a) $\displaystyle\lim_{t\to 0}\frac{\sqrt{a+t}-\sqrt{a-t}}{t}$ for $a>0$

   (b) $\displaystyle\lim_{h\to 0}\frac{1/(a+h)^2-1/a^2}{h}$ for $a\neq 0$

4. T/F (with justification) If $\lim_{x\to 2} g(x)=0$ and $\lim_{x\to 2} h(x)=0$ then $\displaystyle\lim_{x\to 2}\frac{g(x)}{h(x)}$ does not exist.

2.5: Continuity

5. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

   (a) Determine the value of a for which $f(x)$ is continuous from the left at 1.

   (b) Determine the value of a for which $f(x)$ is continuous from the right at 1.

   (c) Is there a value of a for which $f(x)$ is continuous at 1? Explain.

6. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $(2, 3)$. Clearly explain your reasoning.

7. Let

$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x > 1 \end{cases}$$

with the value of $f(1)$ to be determined.

(a) Compute $\lim_{x \to 1^-} f(x)$ in terms of k.

(b) Compute $\lim_{x \to 1^+} f(x)$ in terms of k.

(c) Find the values of k and $f(1)$ that make $f(x)$ continuous at $x = 1$.

(d) Using the choice of k and $f(1)$ in part (c), make a graph of $y = f(x)$ for $0 \leq x \leq 2$.

8. The function $f(x)$ is continous on the interval $(-3, 4)$. If we know that $f(-1) = 4$ and $f(3) = 7$, what can we say about the outputs of $f(x)$, i.e. what values does f definitely take and/or not take?

9. T/F (with justification) The function

$$f(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ 1 + \cos x & \text{if } x > 0 \end{cases}$$

has a jump discontinuity at $x = 0$.

10. T/F (with justification) A function that is continuous at a point has to be defined at the point.

11. T/F (with justification) A function that is discontinuous at a point can't be defined at the point.

2.6: Limits at Infinity and Horizontal Asymptotes

12. Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

    (a) $\lim_{x\to\infty} \frac{2x+3}{6x-7}$

    (b) $\lim_{x\to-\infty} \frac{x^3}{\sqrt{6x^4-1}}$

    (c) $\lim_{x\to\infty} \sqrt{x^2+3x} - x$

    (d) $\lim_{x\to\infty} \frac{100000x}{x^3+x}$

(e) $\lim_{x\to\infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$

(f) $\lim_{x\to-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(g) $\lim_{x\to\infty} \sqrt{x} + \sin x$

(h) $\lim_{x\to\infty} \frac{1}{x} + \sin x$

13. Let $f(x) = \dfrac{\sqrt{4x^6+5}}{x^3-1}$.

(a) Compute $\lim_{x\to\infty} f(x)$.

(b) Compute $\lim_{x\to-\infty} f(x)$.

(c) What are the horizontal asymptotes of the graph of $y = f(x)$?

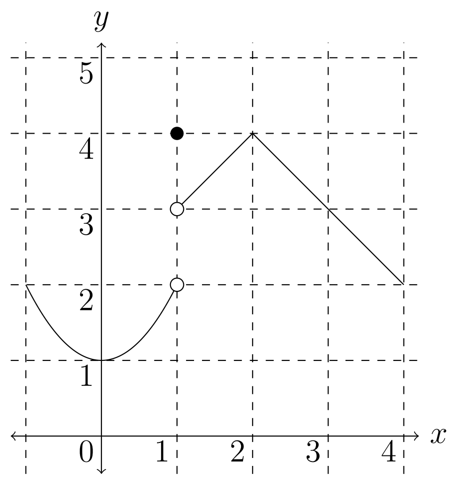
(d) What is the vertical asymptote of the graph of $y = f(x)$?

14. T/F (with justification) The graph of the function $y(x) = 3 + 6e^{-kx}$, with k a positive constant, has a horizontal asymptote $y = 6$.

15. T/F (with justification) If the continuous function $f(x)$ has domain $(-\infty, +\infty)$, then either $\lim_{x\to\infty} f(x)$ exists or $\lim_{x\to\infty} f(x)$ is $\pm\infty$.

Answers to Selected Worksheet Problems

1. (a) Here is a graph of $y = f(x)$ for $-1 \leq x \leq 4$.



(b) From the graph we can evaluate the limits:

(i) 2	(ii) 3	(iii) DNE
(iv) 4	(v) 4	(vi) 4

2. (a) 1.5
   (b) 6
   (c) 3/7
   (d) 4
   (e) DNE
   (f) 4

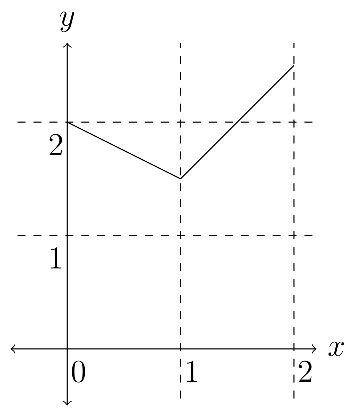
3. (a) $1/\sqrt{a}$
   (b) $-2/a^3$

4. False

5. (a) $a = 2$
   (b) $a = 0$
   (c) No

6. Contact us with questions!

7. (a) $2 - k$.
   (b) $k + 1$.
   (c) $k = 1/2$, $f(1) = 1.5$.
   (d)



8. $f(x)$ must take on every value between 4 and 7 when x is between -1 and 3. It may or may not also take on values ≤ 4 and ≥ 7.

9. True

10. True

11. False

12. (a) $\frac{1}{3}$
    (b) $-\infty$
    (c) $\frac{3}{2}$
    (d) 0
    (e) $\frac{1}{2}$
    (f) -1
    (g) ∞
    (h) DNE

13. (a) 2
    (b) -2
    (c) $y = 2$ and $y = -2$
    (d) $x = 1$

14. False

15. False