

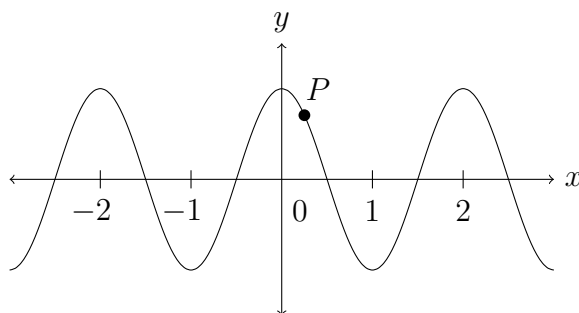
Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

**Solutions should show all of your work, not just a single final answer.**

## 2.1: The Tangent and Velocity Problems

1. The point  $P = (1/4, 1/\sqrt{2})$  lies on the curve  $y = \cos(\pi x)$  where  $x$  is in **radians**, as shown below.



- (a) If  $Q = (x, \cos(\pi x))$  then use your calculator to find the slope of the secant line  $PQ$ , rounded to four digits after the decimal point, for the following values of  $x$ :
- |               |              |
|---------------|--------------|
| (i) 0.24,     | (iv) 0.26,   |
| (ii) 0.249,   | (v) 0.251,   |
| (iii) 0.2499, | (vi) 0.2501. |
- (b) Using the results of part(a), estimate the value of the slope of the tangent line to the curve at  $(1/4, 1/\sqrt{2})$  to three digits after the decimal point.

- (c) Using the estimated slope from part(b), what is an estimate for the equation of the tangent line to the graph of  $y = \cos(\pi x)$  at  $(1/4, 1/\sqrt{2})$ ? Write the final answer in the form  $y = mx + b$  where  $m$  and  $b$  are each rounded to three digits after the decimal point.

2. The displacement of an object on a line, in meters, is  $s = 1 + 2t + \frac{1}{4}t^2$ , where  $t$  is in seconds.

- (a) Find the average velocity in m/sec over each of the following time periods. For parts (i) through (v), *round your answer to three digits after the decimal point*. In part (vi),  $h$  is a nonzero variable and the final answer is in terms of  $h$ .

(i)  $[1, 1.5]$

(ii)  $[1, 1.1]$

(iii)  $[1, 1.01]$

(iv)  $[\cdot 9, 1]$

(v)  $[\cdot 99, 1]$

(vi)  $[1, 1 + h]$  (for  $h > 0$ ) or  $[1 + h, 1]$  (for  $h < 0$ )

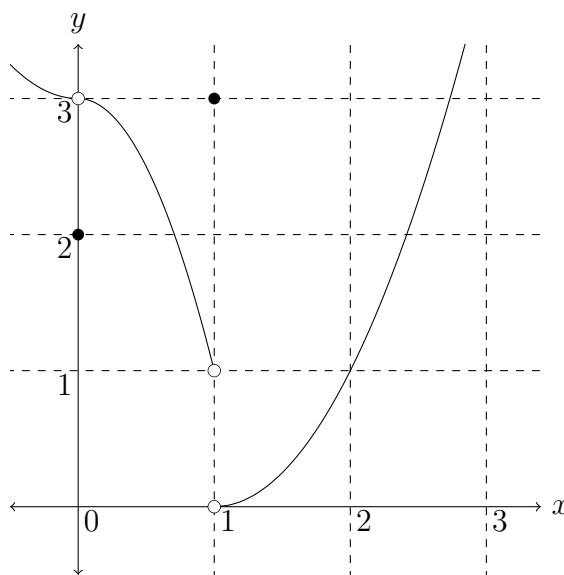
- (b) Use the work in part a to estimate the instantaneous velocity of the object at time  $t = 1$ , in m/sec.

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## 2.2: The Limit of a Function

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3. The graph of  $y = f(x)$  is below. Use it to compute each limit or explain why it doesn't exist.



(a)  $\lim_{x \rightarrow 0^-} f(x)$

(b)  $\lim_{x \rightarrow 1^-} f(x)$

(c)  $\lim_{x \rightarrow 2^-} f(x)$

(d)  $\lim_{x \rightarrow 0^+} f(x)$

(e)  $\lim_{x \rightarrow 1^+} f(x)$

(f)  $\lim_{x \rightarrow 2^+} f(x)$

(g)  $\lim_{x \rightarrow 0} f(x)$

(h)  $\lim_{x \rightarrow 1} f(x)$

(i)  $\lim_{x \rightarrow 2} f(x)$

(j)  $f(0)$

(k)  $f(1)$

(l)  $f(2)$

4. Determine whether the following limits are finite,  $\infty$ , or  $-\infty$ . If the limit does not exist for any other reason, write DNE with a justification.

(a)  $\lim_{x \rightarrow 1} \frac{\sqrt{x}}{2(x-1)^2}$

(b)  $\lim_{x \rightarrow 1^+} \frac{x-2}{x-1}$

(c)  $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2}$

5. T/F (with justification) The line  $x = 1$  is a vertical asymptote of the graph of  $y = \frac{x^2 - 1}{x^2 - 2x + 1}$ .

6. T/F (with justification) The line  $x = 1$  is a vertical asymptote of the graph of  $y = \frac{x^2 - 2x + 1}{x^2 - 1}$ .

Answers to Selected Worksheet Problems

1. (a) (i)  $m(.24) = -2.1862$   
(ii)  $m(.249) = -2.2179$   
(iii)  $m(.2499) = -2.2211$   
(iv)  $m(.26) = -2.2560$   
(v)  $m(.251) = -2.2249$   
(vi)  $m(.2501) = -2.2218$   
(b) Approximately  $-2.221$ .  
(c)

$$y = -2.221x + 1.262$$

2. (a) (i) 2.625    (ii) 2.525    (iii) 2.503    (iv) 2.475    (v) 2.498    (vi)  $2.5 + h/4$

(b) The instantaneous velocity is 2.5.

3.    a. 3                      d. 3                      g. 3                      j. 2  
      b. 1                      e. 0                      h. DNE                  k. 3  
      c. 1                      f. 1                      i. 1                      l. 1

4. (a)  $\lim_{x \rightarrow 1} f(x) = \infty$ .  
(b)  $\lim_{x \rightarrow 1^+} f(x) = -\infty$ .  
(c)  $\lim_{x \rightarrow 0} f(x) = -\infty$ .

5. True

6. False