Math 1131Q Prerequisites Worksheet

Name: _____

Discussion Section: _____

Solutions should show all of your work, not just a single final answer.

Review of Algebra and Functions

I. <u>Fractions:</u> Fractions are an integral part of any math class, and you will need to be proficient with them to solve the problems in this course.

1. Rewrite the following sum as a single fraction.

$$\frac{1}{a+b} + \frac{2}{a} - \frac{3}{b}$$

2. Rationalize the denominators of the following expressions. A good strategy is to multiply both the numerator and denominator by the conjugate of the denominator. For example, $2 - \sqrt{7}$ has $2 + \sqrt{7}$ as its conjugate.

   (a) $\dfrac{4}{1-\sqrt{3}}$

   (b) $\dfrac{x-5}{x+\sqrt{5}}$

II. <u>Difference Quotients:</u> A difference quotient often takes one of the following forms:

$$\frac{f(x+h)-f(x)}{h}, \quad \frac{f(a+h)-f(a)}{h}, \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}.$$

Typically in these expressions, x and h are variables with $h \neq 0$ and a is a fixed value or constant. The first form above is most common and will appear soon in this course. When f is a polynomial or rational function, you can tell when you have finished simplifying the expression because the h in the denominator should cancel.

3. Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the given function.

   (a) $f(x) = 1 - x^2$

   (b) $f(x) = \dfrac{1}{x+1}$

III. Interval Notation:

4. Write the following in interval notation. Use the symbol $\cup$ when writing the union of intervals.
   (a) The open interval with endpoints at 2 and 3.

   (b) The closed interval with endpoints at 2 and 3.

   (c) The half-open interval with endpoints at 2 and 3 that contains 2 but not 3.

   (d) The x-values where the function $f(x) = \dfrac{1}{x}$ is defined.

IV. Solving Equations:

5. Solve for x in terms of y: $2y^2x - y^2 - (1 + 3y) = x$.

6. Determine the solutions of $\dfrac{1}{x} - \dfrac{1}{x+2} = 2$.

7. Determine all solutions of $x^3 - a^2x = 0$, where a is a constant.

V. <u>Functions:</u>

8. Determine the domain of the following functions (the domain is the set of all x-values where the function $f(x)$ is defined). Write your answer in interval notation, using $\cup$ if necessary.

   (a) $f(x) = \dfrac{x+4}{x^2 - x - 6}$

   (b) $f(x) = \sqrt{x^2 - 9}$

9. Consider the function $f(x) = \dfrac{x^2 - 4x - 5}{x^2 + 1}$.

   (a) Determine all zeros of f.

   (b) What are the x- and y-intercepts of the graph of this function? Give your answers as ordered pairs (x, y).

   (c) On what interval(s) is $f(x)$ positive? Negative?

VI. Exponential and Logarithmic Functions:

10. Simplify

(a) $\dfrac{2^{5x}}{2^x}$ (b) $e^{2x}e^{-3x}$ (c) $\dfrac{e^{2x}-1}{e^x-1}$ (d) $\sqrt[3]{5^{2x}}$

11. Evaluate $\log_4(1/64)$.

12. Solve for x exactly: (a) $\log_2 x + \log_2(x-2) = 3$ and (b) $\ln x - \ln(x^2) = 5$.

13. Find all $x > 0$ that satisfy $x^{\sqrt{x}} = x\sqrt{x}$. (Hint: write everything in terms of $y = \sqrt{x}$.) Check that your solution(s) work.

VII. Trigonometric Functions:

14. On the unit circle mark off the following angles (in radians):

(a) $\dfrac{\pi}{2}, \pi,$ and $-\dfrac{\pi}{2}$ together (b) $\dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$ together.

15. Evaluate the following.

(a) $\sin\left(\frac{7\pi}{2}\right)$

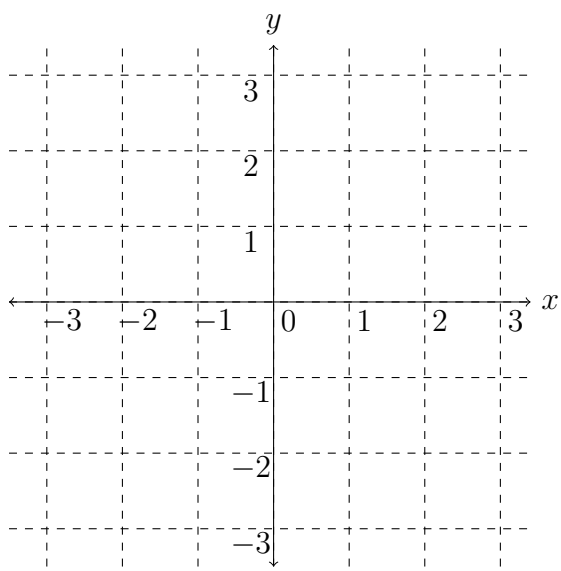
(b) $\cos\left(\frac{-\pi}{2}\right)$

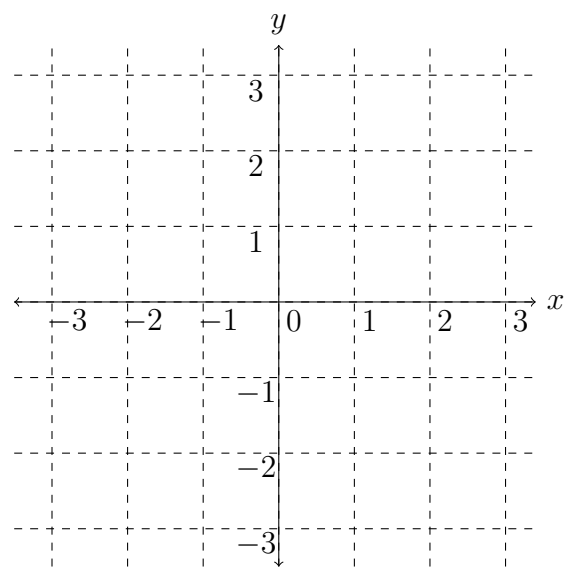
(c) $\sin\left(101\pi\right)$

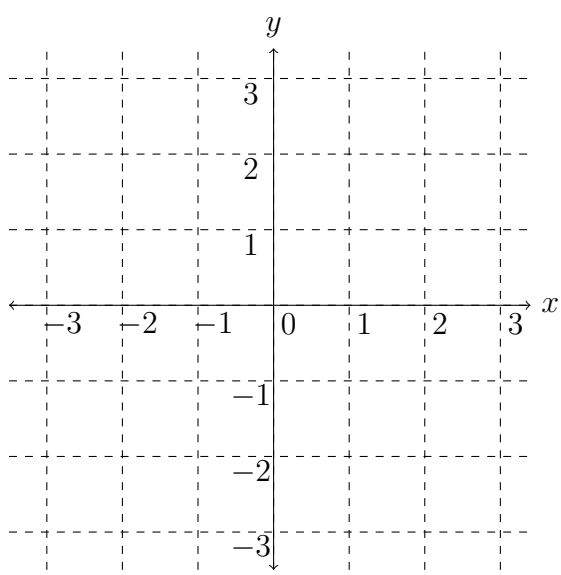
(d) $\sin\left(\frac{\pi}{2} + 2k\pi\right)$ where k is an integer.

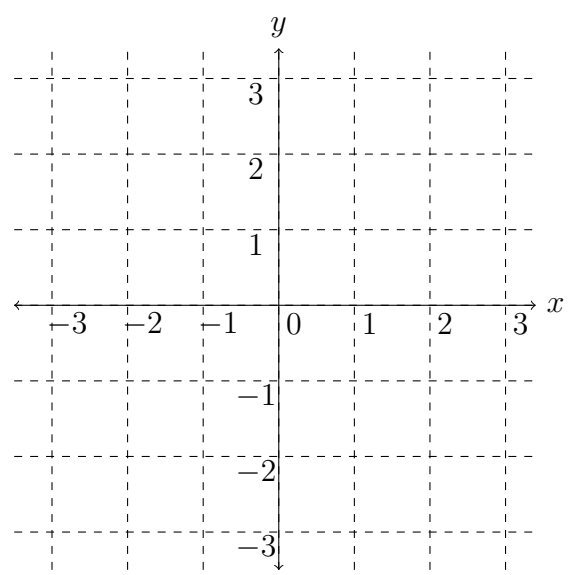
VIII. Graphing Equations: Make sure to re-familiarize yourself with the graphs of common functions and equations, e.g., lines, parabolas, basic cubics, circles, square roots, absolute value, piecewise functions, etc.

16. Using the axes provided below, sketch a graph of each of the following functions.
    - (a) $y = (x-1)^3$
    - (b) $f(x) = \sqrt{4 - x^2}$
    - (c) $y = \dfrac{1}{x-1}$
    - (d) $g(x) = \begin{cases} 1 - x, & x \le 0 \\ x^2 - 1, & x > 0 \end{cases}$









Answers to Selected Worksheet Problems

1. $\dfrac{2b^2 - 3a^2}{ab(a+b)}$

2. (a) $-2(1+\sqrt{3})$

   (b) $\dfrac{(x-5)(x-\sqrt{5})}{x^2-5}$

3. (a) $-2x - h$.

   (b) $\dfrac{-1}{(x+h+1)(x+1)}$.

4. (a) $(2,3)$, (b) $[2,3]$, (c) $[2,3)$, (d) $(-\infty,0)\cup(0,\infty)$.

5. $x = \dfrac{y^2+3y+1}{2y^2-1}$.

6. $x = (-2 \pm \sqrt{8})/2 = -1 \pm \sqrt{2}$.

7. $x = 0$ or $x = \pm a$.

8. (a) $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$.

   (b) $(-\infty,-3]\cup[3,\infty)$.

9. (a) $x = -1$ or $x = 5$

   (b) $x-$intercept:$(-1,0)$, $(5,0)$.
   $y-$intercept: $(0,-5)$.

   (c) The diagram below indicates where $f(x)$ is positive, negative, and zero.

sign$(f(x))$	$+$	0	$-$	0	$+$
x		-1		5	

10. (a) 2^{4x}, (b) e^{-x}, (c) $e^x + 1$, (d) $5^{2x/3}$.

11. -3.

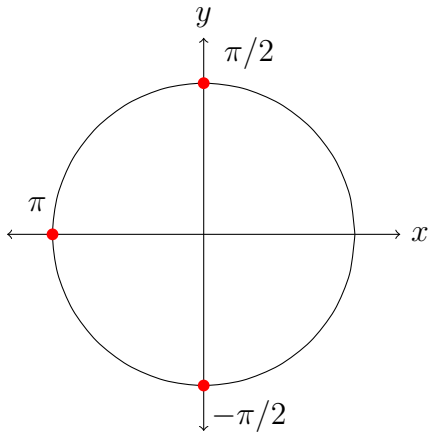
12. (a) $x = 4$.

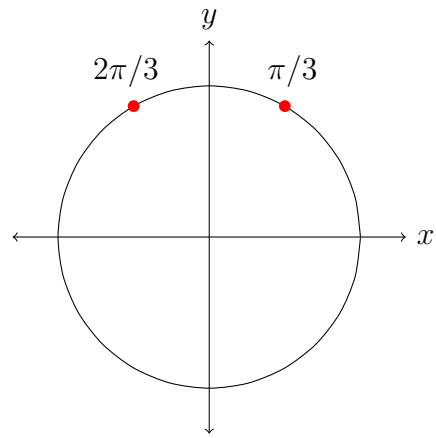
    (b) $t = 1/e^5$.

13. $x = 1$ and $9/4$.

    Both of these choices, $x = 1$ and $x = 9/4$, satisfy the original equation: if $x = 1$ then $x^{\sqrt{x}} = 1^1 = 1$ and $x\sqrt{x} = 1 \cdot 1 = 1$, and if $x = 9/4$ then $x^{\sqrt{x}} = (9/4)^{3/2} = \sqrt{9/4}^3 = (3/2)^3 = 27/8$ and $x\sqrt{x} = (9/4)(3/2) = 27/8$.

14. Here are pictures of the angles.





15. (a) -1.
    (b) 0.
    (c) 0.
    (d) 1.

16. In Figures 1 through 4 are the graphs. See the captions.

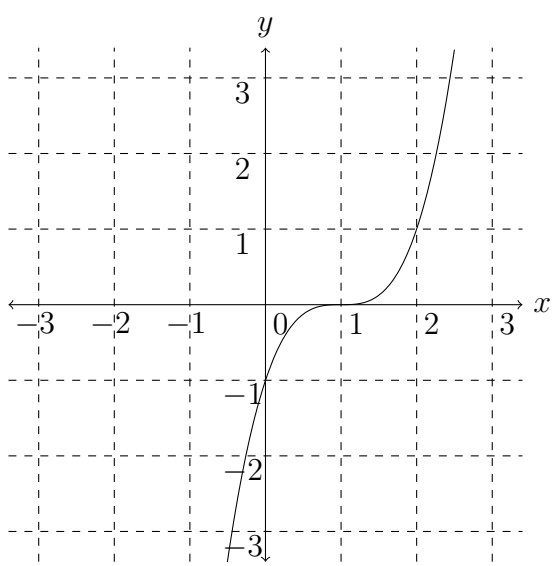


Figure 1: Graph of $y = (x-1)^3$.

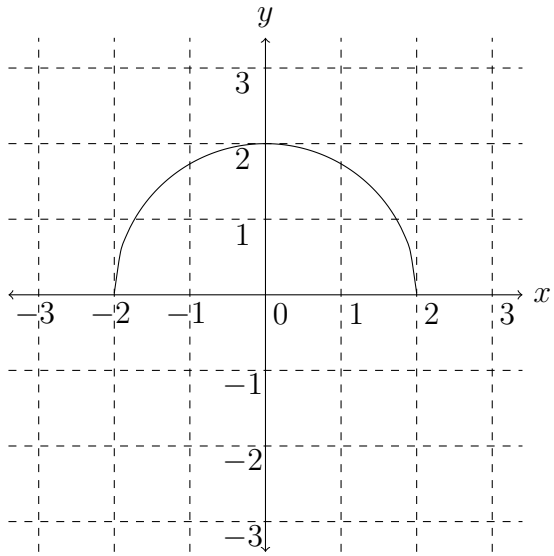


Figure 2: Graph of $f(x) = \sqrt{4 - x^2}$.

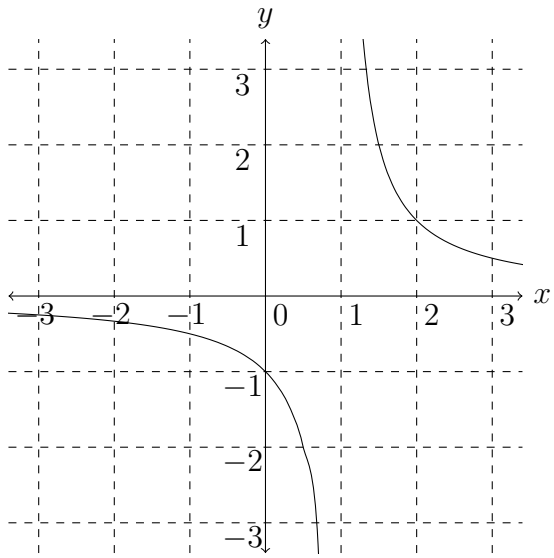


Figure 3: Graph of $y = \dfrac{1}{x-1}$.

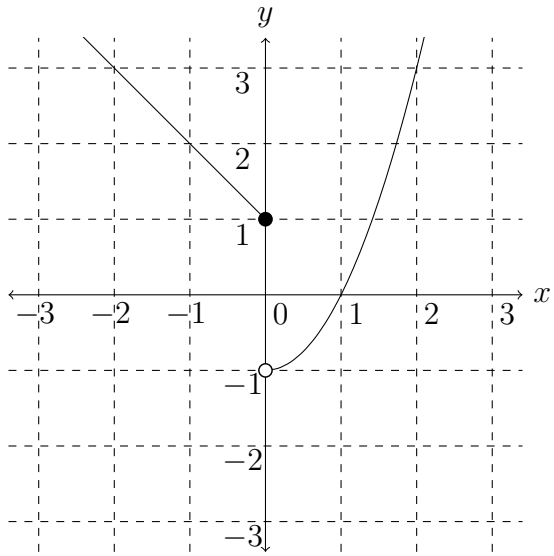


Figure 4: Graph of $g(x) = \begin{cases} 1 - x, & x \leq 0 \\ x^2 - 1, & x > 0 \end{cases}$.