Math 1131 Week 13 Worksheet

Name: ______________________

Discussion Section:____________

Solutions should show all of your work, not just a single final answer.

5.4: Indefinite Integrals and the Net Change Theorem

1. Evaluate each indefinite integral.

   (a) $\displaystyle\int \left(x^2 + \frac{1}{x^3}\right)\, dx$

   (b) $\displaystyle\int \frac{x^3 - 2\sqrt{x}}{x}\, dx$

   (c) $\displaystyle\int \frac{1}{\cos^2 x} dx$ (Hint: $\dfrac{1}{\cos x} = \sec x$)

2. Water is released into a tank at the rate $r(t) = 5 + \sqrt{t}$ ft^3/min at time t (in minutes). At time $t = 1$, there is 12 ft^3 of water in the tank.

   (a) Evaluate $\displaystyle\int_1^6 r(t)\,dt$, rounding your answer to two decimal places.

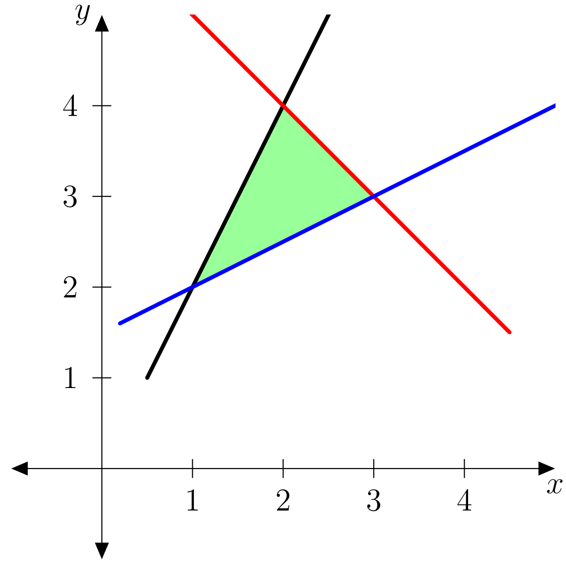
   (b) In the context given above, what does the value in part (a) tell us?

   (c) Determine the volume of water in the tank at time $t = 6$.

3. T/F (with justification) $\displaystyle\int \cos(x^2)\,dx = \sin(x^2) + C$.

6.1: Areas Between Curves

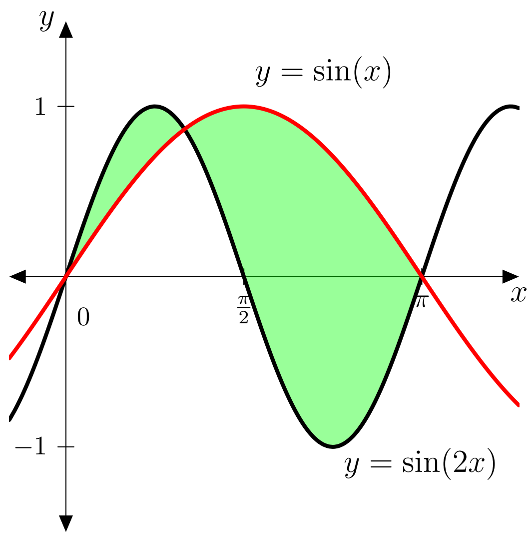
4. We want to find the area of the triangle with vertices $(1, 2)$, $(2, 4)$, and $(3, 3)$, by calculus.



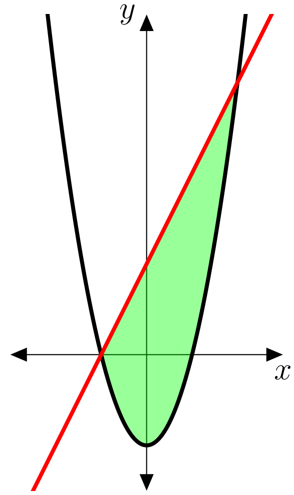
(a) Find equations for the three lines connecting the vertices.

(b) Use the equations in part (a) to express the area of the triangle in terms of integrals, and then compute the area.

5. Find the area of the regions below, between $y = \sin x$ and $y = \sin(2x)$ for $0 \leq x \leq \pi$. (Hint: To find the exact coordinates of the point where the graphs cross, recall that $\sin(2x) = 2\sin x \cos x$.)



6. We want to find the area of the region bounded by $y = 2x + 4$ and $y = x^2 - 4$.



(a) Determine the coordinates of the points where the line and parabola intersect.

(b) Express the area as an integral with respect to x.

(c) Express the area as an integral with respect to y.

(d) Explain which of (b) or (c) is simpler to compute, and use the simpler one to find the area. Simplify your final answer.