University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 4

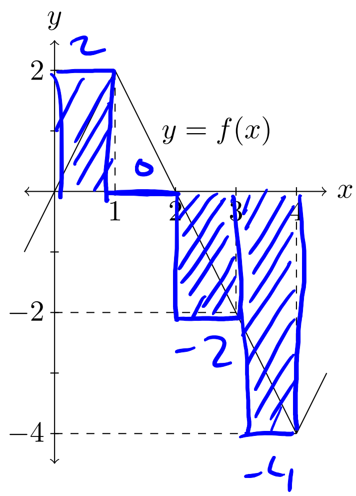
Sections Covered: 5.2-5.5 and 6.1-6.2, plus some previous material

Read This First!

- These practice problems are NOT sufficient review for the exam and do not represent the exact length of the exam. You should also use other resources, such as the textbook, worksheets, and Paul's Online notes to find further practice problems on topics that you have struggled with (or that you have trouble with on the practice problem set).
- Use these practice problems, in addition to other course materials, as a guide to determine what you need to study more deeply.
- The exam will be held during the University-scheduled Final Exam period - but it is the same length (up to 15 questions) and is not cumulative like a traditional Final Exam.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, please carefully check all of your final answers. The submitted letter answers are the **ONLY** place that counts as your official answers.
- You may **NOT** use a calculator or any other references on the exam, and **you are expected to work independently.**

1. If we use a right endpoint approximation with four subintervals (i.e., R_4), then what is the resulting approximation for [1]

$$\int_0^4 f(x)\,dx?$$



(A) 0 (B) −2 (C) $\boxed{-4}$

(D) 2 (E) −3

$2+0-2-4=-4$

2. Evaluate the definite integral $\int_{-1}^{1} (x^2 + 2x + 1)\,dx$. $= \left[\frac{x^3}{3}+x^2+x\right]_{-1}^{1}$ [1]

(A) 3 (B) 2 (C) 2/3

(D) 0 (E) $\boxed{8/3}$

$= \left(\frac{1}{3}+1+1\right)-\left(-\frac{1}{3}+1-1\right)$

$= \frac{2}{3}+2=\frac{8}{3}$

3. Assume that $\int_{-2}^{3} f(x)\,dx = 4$ and $\int_{-2}^{5} f(x)\,dx = 3$. What is the value of $\int_{5}^{3} (f(x)+1)\,dx$? [1]

$\boxed{\text{(A) } -1}$ (B) 2 (C) 1

(D) -2 (E) 3

$$\int_{-2}^{5} f(x)\,dx = \int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx$$

$$\rightarrow \quad 3 = 4 + \int_{3}^{5} f(x)\,dx$$

$$\rightarrow \int_{3}^{5} f(x)\,dx = -1 \rightarrow \int_{5}^{3} f(x)\,dx = 1$$

$$\rightarrow \int_{5}^{3} (f(x)+1)\,dx = \int_{5}^{3} f(x)\,dx + \int_{5}^{3} dx$$

$$= 1 + x\Big|_{5}^{3}$$

$$= 1 + (3-5) = -1$$

4. Given the function $f(x)$ below, determine $f'(2)$. [1]

$$f(x) = \int_{1}^{x^2} \frac{1}{t^2+1}\,dt.$$

$\boxed{\text{(A) } \frac{4}{17}}$ (B) $\frac{1}{17}$ (C) $\frac{2}{5}$

(D) $\frac{2}{17}$ (E) $\frac{4}{5}$

$f(x) = g(u)$ with $u(x) = x^2$ and $g(x) = \int_{1}^{x} \frac{1}{t^2+1}\,dt$

So $f'(x) = g'(u)\frac{du}{dx} = \frac{1}{u^2+1} \cdot 2x$

$$\rightarrow f'(x) = \frac{2x}{(x^2)^2+1}$$

$$f'(2) = \frac{2 \cdot 2}{4^2+1} = \frac{4}{17}$$

5. If $w'(t) = \frac{\ln(t)}{t}$ is the rate of growth of a child in pounds per year, what is the value of $\int_5^{10} w'(t)\,dt$ and what does it mean? [1]

$u = \ln t$

$dt = \frac{1}{t}dt$

$t=5 \to u = \ln 5$

$t=10 \to u = \ln 10$

So

(A) $\ln(10) - \ln(5)$, the child weighs $\ln(10) - \ln(5)$ pounds more at age 10 than at age 5.

(B) $\frac{75}{2}$, the child gains weight at a rate of $\frac{75}{2}$ pounds per year from agae 5 to age 10.

$\boxed{\text{(C) } \frac{[\ln(10)]^2 - [\ln(5)]^2}{2}\text{, the child weighs } \frac{[\ln(10)]^2 - [\ln(5)]^2}{2} \text{ lbs more at age 10 than at age 5.}}$

(D) $\frac{[\ln(10)]^2 - [\ln(5)]^2}{2}$, the child gains weight at a rate of $\frac{[\ln(10)]^2 - [\ln(5)]^2}{2}$ pounds per year from agae 5 to age 10.

(E) $\frac{75}{2}$, the child weighs $\frac{75}{2}$ pounds more at age 10 than at age 5.

$$\int_5^{10} w'(t)\,dt = \int_5^{10} \frac{\ln t}{t}\,dt = \int_{\ln 5}^{\ln 10} u\,du = \left.\frac{u^2}{2}\right|_{\ln 5}^{\ln 10} = \frac{(\ln 10)^2}{2} - \frac{(\ln 5)^2}{2},$$

this represents a net change in weight, so the child weighed $\frac{(\ln 10)^2}{2} - \frac{(\ln 5)^2}{2}$ lbs more at $t=10$ than at $t=5$.

6. Evaluate $\int_0^{\pi/4} \frac{1+\cos^2(x)}{\cos^2(x)}\,dx$. [1]

(A) $\frac{1}{2}$ (B) $\ln(1/2)$ (C) $\frac{\pi}{4}$

$\boxed{\text{(D) } 1 + \frac{\pi}{4}}$ (E) $\frac{1}{3}$

$$= \int_0^{\pi/4} \left(\frac{1}{\cos^2 x} + 1\right) dx = \int_0^{\pi/4} (\sec^2 x + 1)\,dx$$

$$= \Big[\tan x + x\Big]_0^{\pi/4} = \tan\frac{\pi}{4} + \frac{\pi}{4} - (\tan 0 + 0)$$

$$= 1 + \frac{\pi}{4} - (0+0)$$

$$= 1 + \frac{\pi}{4}$$

7. Evaluate $\int_0^1 (x^{10} + 10^x)\,dx$. [1]

(A) $10 + 9\ln(10)$ (B) $\frac{100}{11}$ (C) $\boxed{\frac{1}{11} + \frac{9}{\ln(10)}}$

(D) $\frac{12}{11}$ (E) 10

$$\left[\frac{x^{11}}{11} + \frac{10^x}{\ln 10}\right]_0^1 = \frac{1}{11} + \frac{10}{\ln 10} - \left(\frac{0}{11} + \frac{1}{\ln 10}\right)$$

$$= \frac{1}{11} + \frac{9}{\ln 10}$$

8. Evaluate $\int \left(\frac{1+r}{r}\right)^2 dr$. [1]

(A) $\frac{-\left(\frac{1+r}{r}\right)^3}{3r^2} + C$ (B) $\frac{1}{r} - \frac{2}{r^2} + r + C$ (C) $\frac{\left(\frac{1+r}{r}\right)^3}{3} + C$

(D) $-\frac{1}{r} + r + C$ (E) $\boxed{-\frac{1}{r} + 2\ln|r| + r + C}$

$$= \int \left(\frac{1}{r} + 1\right)^2 dr$$

$$= \int \left(\frac{1}{r^2} + \frac{2}{r} + 1\right) dr$$

$$= -\frac{1}{r} + 2\ln|r| + r + C$$

9. Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$. [1]

(A) $2e^{\sqrt{x}} + C$ (B) $2e^{\sqrt{x}}\sqrt{x} + C$ (C) $e^{\sqrt{x}} + C$

(D) $4e^{\sqrt{x}}\sqrt{x} + C$ (E) $\frac{e^{\sqrt{x}}(\sqrt{x}-1)}{2x} + C$

let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx \to \frac{1}{\sqrt{x}}dx = 2du$

So $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int e^u \cdot 2du$

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

10. Evaluate $\int_5^{10} \frac{dt}{(t-4)^2}$. [1]

(A) $-\frac{5}{6}$ (B) $\frac{1}{10}$ (C) $\frac{5}{3}$

(D) $\frac{5}{6}$ (E) $-\frac{1}{10}$

$u = t-4$

$du = dt$

$t=5 \to u=1$

$t=10 \to u=6$

so $\int_5^{10} \frac{dt}{(t-4)^2} = \int_1^6 \frac{1}{u^2}du$

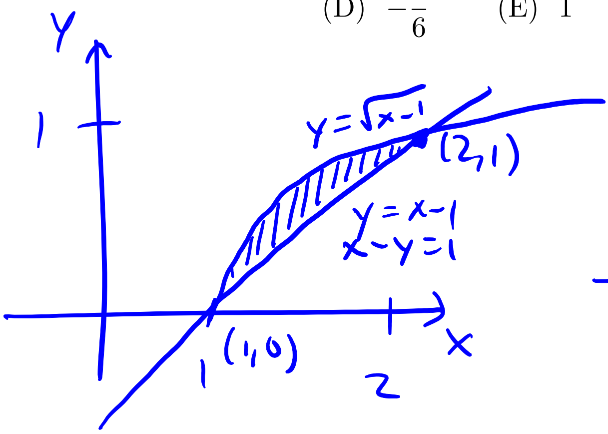
$= -\frac{1}{u}\Big|_1^6$

$= -\frac{1}{6} + 1 = \frac{5}{6}$

11. Determine the area of the region bounded by $y = \sqrt{x-1}$ and $x - y = 1$. [1]

(A) $-\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{2}{3}$

(D) $-\frac{1}{6}$ (E) 1



$$\int_1^2 \left[\sqrt{x-1} - (x-1)\right] dx$$

$u = x-1$ $\quad x=1 \to u=0$
$du = dx$ $\quad x=2 \to u=1$

$$\to \int_0^1 (\sqrt{u} - u)\, du$$

$$= \left[\frac{2}{3}u^{3/2} - \frac{u^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2}$$

$$= \frac{1}{6}$$

12. Which expression could be used to calculate the area of the triangle with vertices $(0,0)$, $(3,1)$, and $(1,2)$? [1]

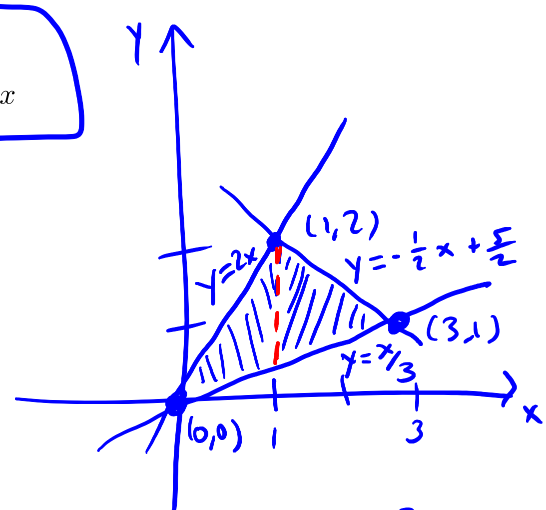
(A) $\int_0^1 \left[2x - \frac{x}{3}\right] dx + \int_1^3 \left[-\frac{5}{6}x + \frac{5}{2}\right] dx$

(B) $\int_0^3 \left[2x - \frac{x}{3}\right] dx$

(C) $\int_0^3 \left[-\frac{5}{6}x + \frac{5}{2}\right] dx$

(D) $\int_0^1 (-x)\, dx + \int_1^3 \left[-\frac{7}{2}x + \frac{5}{2}\right] dx$

(E) $\int_0^3 \left[-\frac{7}{2}x + \frac{5}{2}\right] dx$



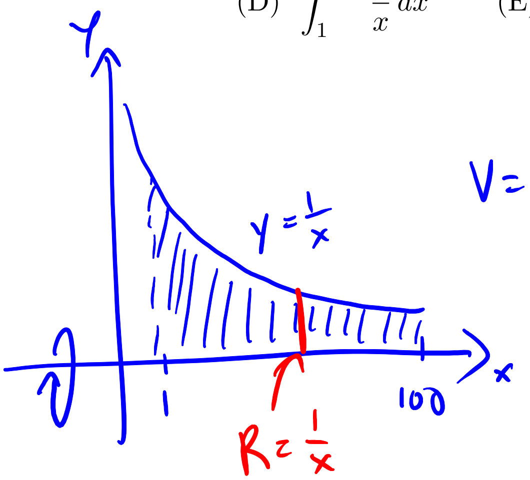
$$\int_0^1 \left[2x - \frac{x}{3}\right] dx + \int_1^3 \left[-\frac{1}{2}x + \frac{5}{2} - \frac{x}{3}\right] dx$$

$$= \int_0^1 \left(2x - \frac{x}{3}\right) dx + \int_1^3 \left(-\frac{5}{6}x + \frac{5}{2}\right) dx$$

13. Write an integral that could be used to calculate the volume of the solid obtained by rotating the region under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = 100$ around the x-axis. [1]

(A) $\int_1^{100} \frac{\pi}{x}\,dx$ (B) $\int_1^{100} \frac{1}{x^2}\,dx$ (C) $\int_1^{100} \frac{\pi}{x^2}\,dx$

(D) $\int_1^{100} \frac{1}{x}\,dx$ (E) $\int_1^{100} \left[\frac{\pi}{x^2} - \frac{1}{x^2}\right] dx$



$V = \int_1^{100} \pi R^2\,dx$

$= \int_1^{100} \frac{\pi}{x^2}\,dx$

14. Find the volume of the solid whose base is this region bounded by the curves $y = 1/x$, $y = 0$, $x = 1$ and $x = 100$ and whose cross-sections perpendicular to the x-axis are right triangles whose height (shorter leg) is half their base (longer leg). [1]

(A) $\int_1^{100} \frac{1}{x^2}\,dx$ (B) $\int_1^{100} \frac{1}{x}\,dx$ (C) $\int_1^{100} \frac{1}{2x^2}\,dx$

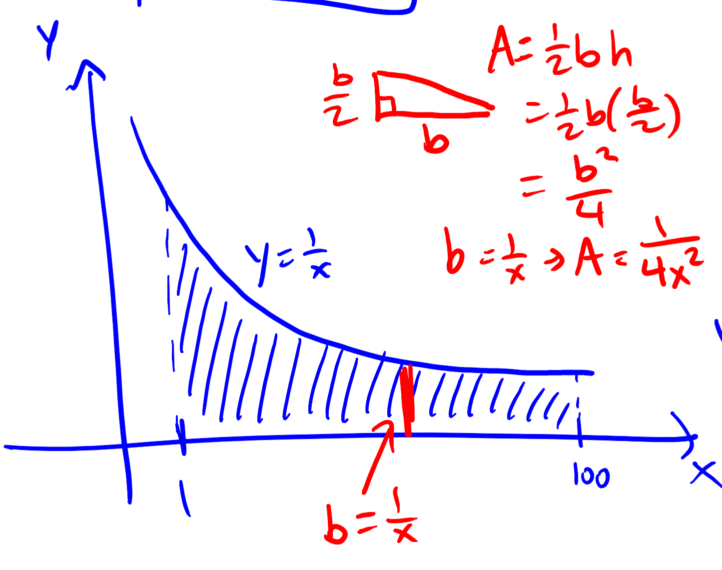
(D) $\int_1^{100} \frac{1}{4x^2}\,dx$ (E) $\int_1^{100} \frac{1}{4x}\,dx$

$A = \frac{1}{2}bh$

$= \frac{1}{2}b\left(\frac{b}{2}\right)$

$= \frac{b^2}{4}$

$b = \frac{1}{x} \Rightarrow A = \frac{1}{4x^2}$

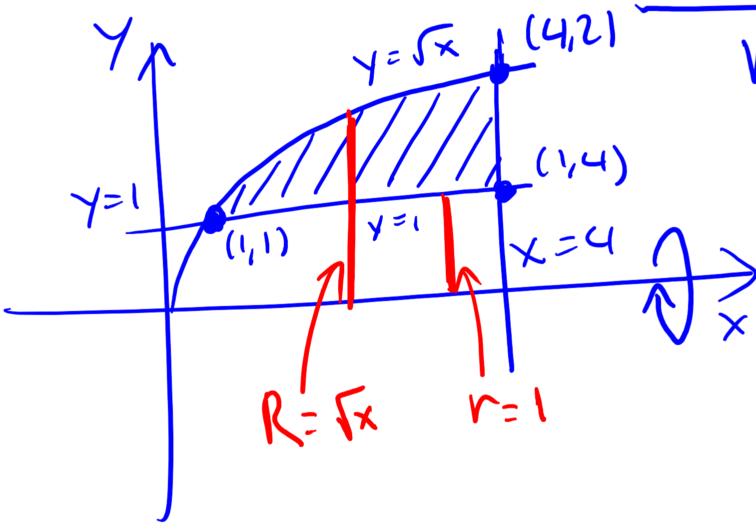


$V = \int_1^{100} \frac{1}{4x^2}\,dx$

15. Write an integral that could be used to calculate the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}, y = 1$ and $x = 4$ around the x-axis. [1]

(A) $\int_1^4 \pi(\sqrt{x}-1)^2\,dx$ (B) $\int_1^4 (\sqrt{x}-1)\,dx$ (C) $\int_1^4 (x-1)\,dx$

(D) $\int_1^4 \pi(\sqrt{x}-1)\,dx$ (E) $\int_1^4 \pi(x-1)\,dx$



$$V = \int_1^4 \pi \left[R^2 - r^2\right] dx$$

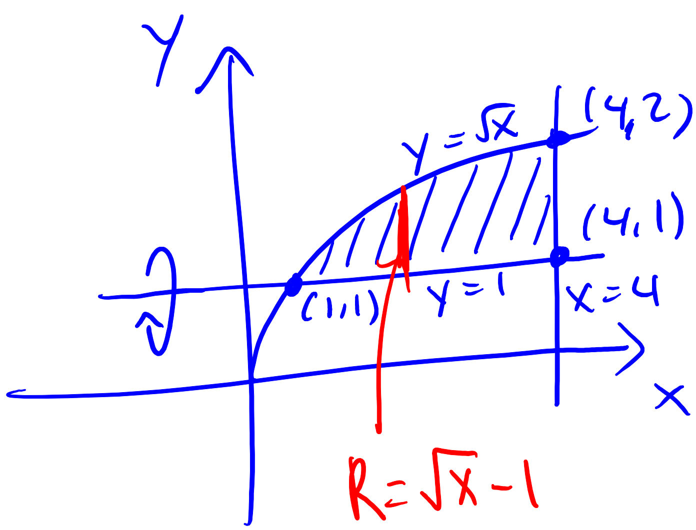
$$= \int_1^4 \pi \left[(\sqrt{x})^2 - 1^2\right] dx$$

$$= \int_1^4 \pi (x-1)\,dx$$

16. Write an integral that could be used to calculate the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}, y = 1$ and $x = 4$ around the line $y = 1$. [1]

(A) $\int_1^4 \pi(\sqrt{x}-1)^2\,dx$ (B) $\int_1^4 (\sqrt{x}-1)\,dx$ (C) $\int_1^4 (x-1)\,dx$

(D) $\int_1^4 \pi(\sqrt{x}-1)\,dx$ (E) $\int_1^4 \pi(x-1)\,dx$



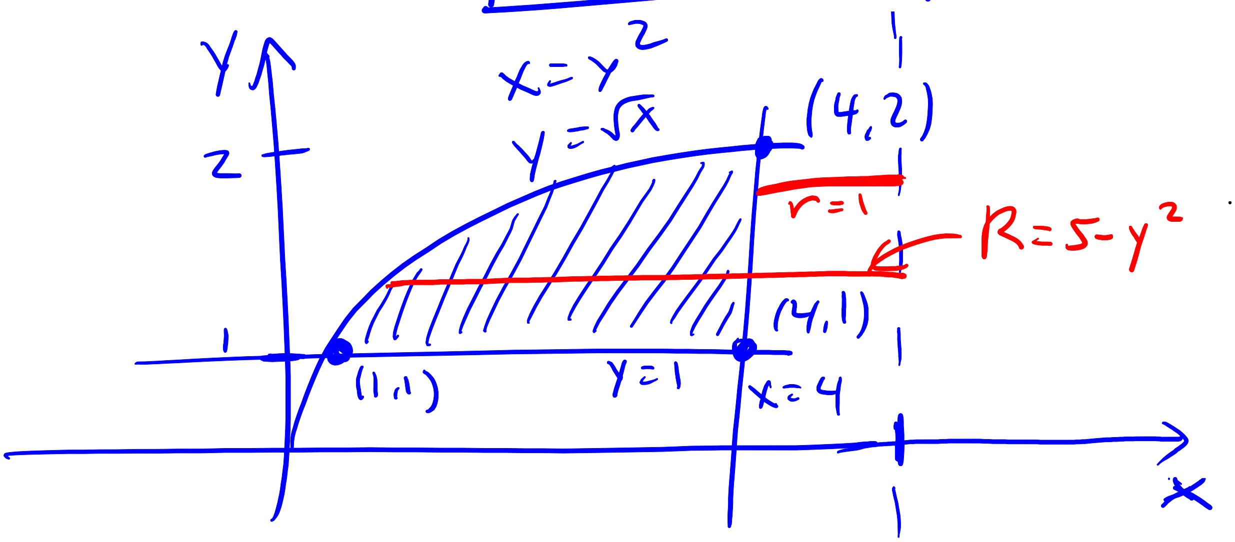
$$V = \int_1^4 \pi R^2\,dx$$

$$= \int_1^4 \pi (\sqrt{x}-1)^2\,dx$$

17. Write an integral that could be used to calculate the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 1$ and $x = 4$ around the line $x = 5$. [1]

(A) $\int_1^2 \pi(4 - y^2)\,dy$ (B) $\int_1^2 \pi(4 - y^2)^2\,dy$ (C) $\int_1^4 \pi(5 - \sqrt{x})^2\,dx$

(D) $\int_1^4 \pi(5 - x)\,dx$ (E) $\int_1^2 \pi\left[(5 - y^2)^2 - 1\right]\,dy$



$$V = \int_{y=1}^{y=2} \pi\left(R(y)^2 - r(y)^2\right) dy$$

$$= \int_1^2 \pi\left[(5-y^2)^2 - 1\right] dy$$