University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 3

Sections Covered: 4.1, 4.2, 4.3, 4.4, 4.7, 4.8, 4.9, and 5.1, plus some previous material

Read This First!

- These practice problems are NOT sufficient review for the exam and do not represent the exact length of the exam. You should also use other resources, such as the textbook, worksheets, and Paul's Online notes to find further practice problems on topics that you have struggled with (or that you have trouble with on the practice problem set).

- Use these practice problems, in addition to other course materials, as a guide to determine what you need to study more deeply.

- The exam will be 50 minutes during your regular discussion section meeting.

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.

- On the exam, please carefully check all of your final answers. The submitted letter answers are the **ONLY** place that counts as your official answers.

- You may **NOT** use a calculator or any other references on the exam, and **you are expected to work independently.**

1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

   (A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ $\boxed{\text{(B) } x_{n+1} = (x_n^2 + 7)/(2x_n)}$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

   (D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x_n) = x_n^2 - 7$

$f'(x_n) = 2x_n$

$\rightarrow x_{n+1} = \cancel{x_n} - \frac{x_n^2 - 7}{2x_n}$

$= \frac{2x_n^2 - (x_n^2 - 7)}{2x_n}$

$= \underline{\frac{x_n^2 + 7}{2x_n}}$

2. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

   (A) 1/2 $\boxed{\text{(B) } 19/6}$ (C) 15/4

   (D) 12/7 (E) 17/6

$f(x_1) = f(3) = 3^2 - 10 = -1$

$f'(x) = 2x \rightarrow f'(x_1) = f'(3) = 2 \cdot 3 = 6$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{(-1)}{6}$

$= \frac{18}{6} + \frac{1}{6} = \underline{\frac{19}{6}}$

3. Which of the following is the absolute maximum value of the function $f(x) = \dfrac{x}{x^2 + 4}$ on the interval $[0, 4]$?

   (A) $\frac{1}{8}$ (B) $\frac{1}{5}$ $\boxed{\text{(C) } \frac{1}{4}}$

   (D) $\frac{1}{2}$ (E) 1

$f'(x) = \frac{1 \cdot (x^2+4) - x \cdot 2x}{(x^2+4)^2}$

$= \frac{4 - x^2}{(x^2+4)^2}$

crit #s: $4 - x^2 = 0 \rightarrow x = \pm 2$

(denom. never zero)

only one crit. # in $[0,4]$: $x = 2$

so $f(0) = 0$

$f(2) = \frac{2}{4+4} = \boxed{\frac{1}{4}}$ $\rightarrow$ (largest) abs. max

$f(4) = \frac{4}{16+4} = \frac{1}{5}$

4. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

   (A) 9 (B) $\sqrt{27}$ $\boxed{\text{(C) } \sqrt{3}}$

   (D) 3 (E) No such value of c exists.

1) f cont. on $[0,3]$ ✓
2) f diff'ble on $(0,3)$ ✓
So MVT applies ✓

$$f'(c) = \frac{f(3) - f(0)}{3 - 0}$$

$$3c^2 = \frac{3^3 - 0^3}{3-0} \to 3c^2 = \frac{27}{3} = 9$$

$$\to c^2 = 3$$

$$c = \pm\sqrt{3}$$

$c = -\sqrt{3}$ not in $[0,3]$, so $\underline{c = \sqrt{3}}$

5. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

   $\boxed{\text{(A) } 1}$ (B) -2 (C) $-2,\ 1$

   (D) $-2,\ \frac{1}{2}$ (E) $-2,\ \frac{1}{2},\ 1$

$$f'(x) = 6x^2 + 6x - 12$$
$$= 6(x^2 + x - 2)$$
$$= 6(x+2)(x-1) = 0 \ (\text{or DNE})$$
$$x = -2, 1$$

2nd deriv. test:

$$f''(x) = 12x + 6$$

$f''(-2) = -24 + 6 = -18$ loc. max at $x = -2$

$f''(1) = 12 + 6 = 18$ loc min at $\underline{x = 1}$

6. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

   (A) 0 (B) 1 $\boxed{\text{(C) } 2}$

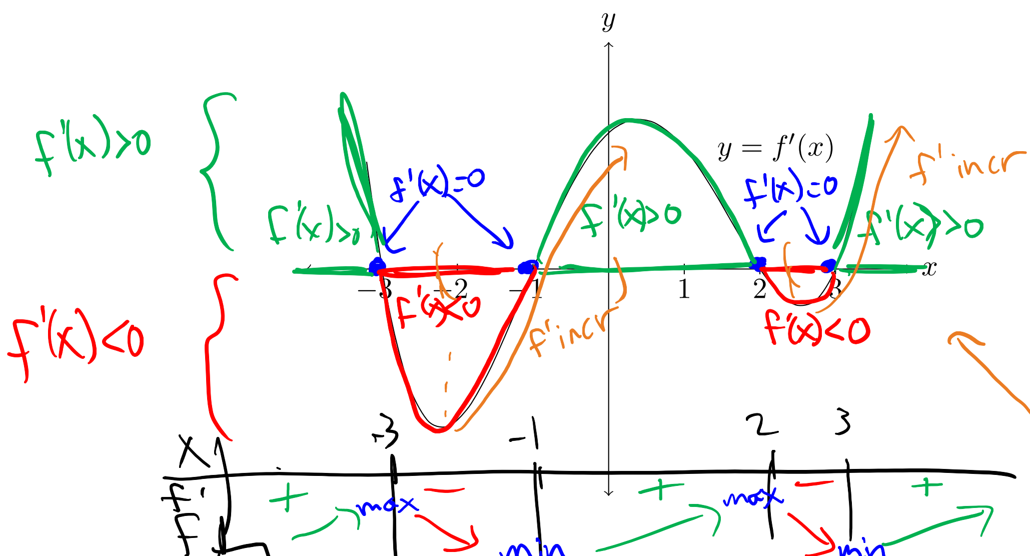
   (D) 3 (E) 4

$$f'(x) = 4x^3 - 16x$$
$$f''(x) = 12x^2 - 16 = 0 \ (\text{or DNE})$$
$$12x^2 = 16$$
$$x^2 = \frac{4}{3}$$
$$x = \pm\frac{2}{\sqrt{3}}$$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	−	+
f			

IP 1 IP 2

7. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



(A) Local maxima at −3 and 2 and local minima at −1 and 3

(B) Local maxima at −1 and 3 and local minima at −3 and 2

(C) Local maxima at −1 and 3 and local minimum at 2

(D) Local maxima at −3 and 2 and local minimum at −1

(E) None of the above

8. Referring to the same graph of the derivative in question 7, at approximately what x-value(s) is $f(x)$ concave up? need $f''>0$, so f' increasing (i.e. $(f')'>0$).

2 intervals: $(-2.1, 0.8)$ and $(2.6, \infty)$

(A) $x < -1$ and $x > 1.5$

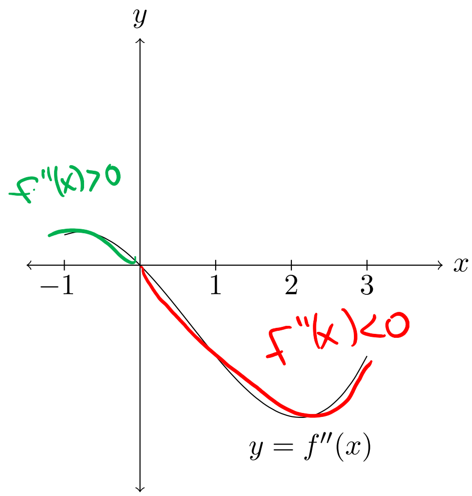
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

9. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. f''>0 here ✓

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. f''<0 here ✓

(C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to − here ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to − here (i.e., f' changes ↗ to ↘ here) ✓

$\boxed{\text{(E)}}$ All of the above.

10. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only $\boxed{\text{(B) } (1, 2) \text{ only}}$ (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$ (~~DNE~~)

$x = 1, 2$

x	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
f''	+	−	+
f	∪	∩	∪
		conc. down	

11. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}. = \text{"}\frac{0}{0}\text{"}$$ use L'Hospital's rule.

$\boxed{\text{(A) } +\infty}$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cos x}{2x}$$

$$= \frac{\text{about } 1}{\text{small } + \to 0} = \underline{\underline{+\infty}}$$

12. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}. = \text{"}\frac{0}{0}\text{"}$$ $\to$ use L'Hospital's.

$\boxed{\text{(A) } 0}$ (B) 1 (C) $+\infty$

(D) -1 (E) 1/2

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{L'H}{=} \lim_{x\to \frac{\pi}{2}} \frac{-\cos x}{-\sin x}$$

$$= \frac{-0}{-1} = \underline{\underline{0}}$$

13. Determine the number of inflection points of the graph of $y = x^2 - \frac{1}{x}$ on its domain.

(A) 0 $\boxed{\text{(B) } 1}$ (C) 2 (D) 3 (E) 4

$$y' = 2x + \frac{1}{x^2} \to y'' = 2 - \frac{2}{x^3} = 0 \text{ or undefined}$$

$y'' = 0$:

$2 = \frac{2}{x^3}$

$x^3 = 1$

$x = 1$

y'' undef.:

$x = 0$

x	$(-\infty, 0)$	asymptote	$(0, 1)$	IP	$(1, \infty)$
y''	+		−		+
y	∪		∩		∪

14. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 $\boxed{\text{(C) 20, 40}}$

(D) 26, 27 (E) None of the above

maximize

$P = xy$ with $y + 2x = 80 \rightarrow y = 80 - 2x$

$P(x) = x(80-2x) = 80x - 2x^2 \rightarrow$ maximize on $(0,40)$

$P'(x) = 80 - 4x = 0$ (or ~~DNE~~)

$80 = 4x$

$\underline{x = 20} \rightarrow y = 80 - 2(20) = \underline{40}$

2nd deriv. test:

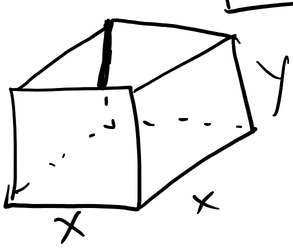
$P''(x) = -4 < 0$

max ✓

15. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?

(A) \$500 (B) \$600 (C) \$1000

$\boxed{\text{(D) \$1200}}$ (E) \$2000



$V = x^2 y = 4000 \rightarrow y = \frac{4000}{x^2}$

Cost $= (\$1)$(total area of bottom + sides)

$= x^2 + 4xy$

$\rightarrow C(x) = x^2 + 4x\left(\frac{4000}{x^2}\right) = x^2 + \frac{16000}{x}$; minimize over $(0,\infty)$

$C'(x) = 2x - \frac{16000}{x^2} = 0$ or ~~DNE~~

~~$x = 0$ (not in domain)~~

$\rightarrow 2x = \frac{16000}{x^2}$

$x^3 = 8000$

$\underline{x = 20}$

min. cost $= C(20) = 20^2 + \frac{16000}{20}$

$= 400 + 800 = \underline{1200}$

$C''(x) = 2 + \frac{32000}{x^3} \rightarrow C''(20) > 0 \rightarrow$ minimum ✓

16. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x\to\infty} \frac{f(x)}{x^2} = \frac{\text{"?"}}{\infty} \to \text{L'H applies if } \lim_{x\to\infty} f(x) = \pm\infty$$

(A) $\sin(x)$ (B) e^{-x} (C) $\cos(x)$

$\boxed{\text{(D) } \ln(x)}$ (E) All of the above

~~A)~~ $\lim_{x\to\infty} \sin x$ DNE

~~B)~~ $\lim_{x\to\infty} e^{-x} = 0$

~~C)~~ $\lim_{x\to\infty} \cos x$ DNE

D) $\lim_{x\to\infty} \ln x = \infty$ ✓

17. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

(A) $1 - \ln 2$ (B) 0 $\boxed{\text{(C) } 2 - \ln 5}$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

$$v'(t) = 1 - \frac{2t}{t^2+1} = 0$$

(or DNE: never since $t^2+1>0$ always)

$$1 = \frac{2t}{t^2+1}$$

$$t^2+1 = 2t$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

max out of $v(0), v(1), v(2)$:

$v(0) = 0 - \ln(1) = 0$

$v(1) = 1 - \ln(2) > 0$

$v(2) = 2 - \ln(5) > 0$

which is larger?

x	0	1	2
v'	+		+
v	↗		↗

max @ $x = 2$ $\longrightarrow$ $v(2) = 2 - \ln(5)$ is max value

18. If $f(1) = 9$ and $f'(x) \geq 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

(A) 19 $\boxed{\text{(B) 18}}$ (C) 12

(D) Cannot be determined (E) None of the above

Using mean value theorem: $f'(c) = \frac{f(4)-f(1)}{4-1}$ for some c in $[1,4]$, so:

$$3f'(c) = f(4) - f(1) \rightarrow f(4) = f(1) + 3f'(c)$$
$$= 9 + 3f'(c) \geq 9 + 3(3) = \underline{\underline{18}}$$

19. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0
		min $f''>0$	CBD $f''=0$	min $f''>0$		max $f''<0$	

crit #s, where $f'=0$.
$x = -3, -2, 0, 4$.

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

$\boxed{\text{(C)}}$ Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6

20. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) 0 (B) 1 (C) -2

(D) 2 (E) -1

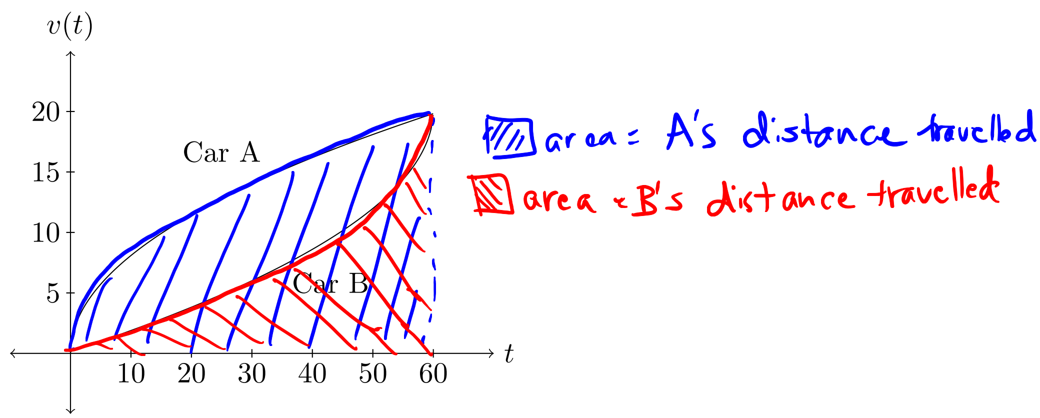
$f'(x) = 2x - \frac{3}{2}x^2 + C, \quad f'(0) = C = -1 \to f'(x) = 2x - \frac{3}{2}x^2 - 1$

$f(x) = x^2 - \frac{1}{2}x^3 - x + D, \quad f(0) = D = 1 \to f(x) = x^2 - \frac{1}{2}x^3 - x + 1$

$\to f(2) = 4 - \frac{1}{2}\cdot 8 - 2 + 1$

$= \underline{\underline{-1}}$

21. Below is the graph of the velocity (measured in ft/sec) over the interval $0 \le t \le 60$ for two cars, Car A and Car B. How do the distances traveled by each compare over this interval?



(A) Car A travelled farther because its speed was increasing the whole time

(B) Car B travelled farther because its speed was increasing the whole time

(C) Car A travelled farther because the area under its velocity curve is larger than B's

(D) Car A and Car B travelled the same distance

(E) Car B travelled farther because it was moving faster at the end

22. Find $f(x)$ if $f'(x) = 3x^2 + \dfrac{2}{x}$ for $x > 0$ and $f(1) = 3$.

(A) $x^3 + 2\ln x$ (B) $x^3 - \dfrac{2}{x^2}$ (C) $x^3 - \dfrac{2}{x^2} + 4$

$\boxed{\text{(D) } x^3 + 2\ln x + 2}$ (E) $x^3 + 2\ln x + 3$

$f(x) = x^3 + 2\cdot\ln|x| + C$

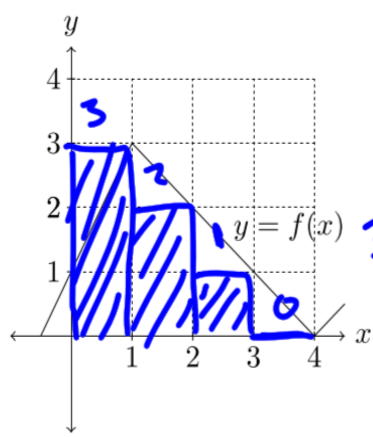
$\rightarrow f(x) = x^3 + 2\ln x + C \quad (x>0)$

$f(1) = 3 \rightarrow 1^2 + \cancel{2\ln 1}^{\,0} + C = 3$

$1 + C = 3 \rightarrow C = 2$

$\rightarrow f(x) = \underline{x^3 + 2\ln x + 2}$

23. If we use a right endpoint approximation with four subintervals (i.e., R_4), then what is the resulting approximation for the area under the curve $y = f(x)$ from $x = 0$ to $x = 4$?



$3 + 2 + 1 + 0 = 6$

(A) 9 (B) 7 (C) 7.5

$\boxed{\text{(D) } 6}$ (E) 6.5