University of Connecticut
Department of Mathematics

Math 1131 — Practice Problems for Exam 2

Sections Covered: 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 3.9, and 3.10, plus some previous material

Read This First!

- These practice problems are NOT sufficient review for the exam and do not represent the exact length of the exam. You should also use other resources, such as the textbook, worksheets, and Paul's Online notes to find further practice problems on topics that you have struggled with (or that you have trouble with on the practice problem set).

- Use these practice problems, in addition to other course materials, as a guide to determine what you need to study more deeply.

- The exam will be 50 minutes during your regular discussion section meeting.

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.

- On the exam, please carefully check all of your final answers. The submitted letter answers are the **ONLY** place that counts as your official answers.

- You may **NOT** use a calculator or any other references on the exam, and **you are expected to work independently.**

1. Evaluate the following limit: [1]

$$\lim_{x\to\infty} \frac{\sqrt{x^3+2}}{x}.$$

(A) $+\infty$ (B) $-\infty$ (C) 0
(D) 1 (E) -1

this equals $\lim_{x\to\infty} \frac{\sqrt{x^3+2}}{\sqrt{x^2}} = \lim_{x\to\infty} \sqrt{x+\frac{2}{x^2}}$

$$\text{"="} \sqrt{\infty+0} \to +\infty$$

2. The function $f(x) = \dfrac{x^2+1}{x^2-4}$ has which of the following? [1]

(A) no vertical or horizontal asymptotes

(B) 1 vertical asymptote and 1 horizontal asymptote

(C) 2 vertical asymptotes and 1 horizontal asymptote

(D) 1 vertical asymptote and 2 horizontal asymptotes

(E) 1 vertical asymptote and no horizontal asymptotes

$f(x) = \dfrac{x^2+1}{(x+2)(x-2)} \to$ 2 vert. asym @ $x = \pm 2$

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \dfrac{1+\frac{1}{x}}{1-\frac{4}{x^2}} = \dfrac{1+0}{1-0} = 1$

$\to$ 1 horiz. asym @ $y=1$

3. If $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ for $x > 0$, then $f'(4)$ is which of the following? [1]

(A) $\frac{5}{4}$ (B) $\frac{3}{4}$ $\boxed{\text{(C) } \frac{3}{16}}$

(D) $\frac{255}{32}$ (E) $\frac{257}{32}$

$$f(x) = x^{1/2} + x^{-1/2} \rightarrow f'(x) = \frac{1}{2}x^{-1/2} + \left(-\frac{1}{2}\right)x^{-3/2}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}\right)$$

$$= \frac{1}{2} \cdot \frac{x-1}{x\sqrt{x}}$$

$$f'(4) = \frac{1}{2} \cdot \frac{4-1}{4\sqrt{4}} = \underline{\underline{\frac{3}{16}}}$$

4. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

(A) 3 (B) 0 (C) 4

(D) 2 $\boxed{\text{(E) } 5}$

$$f'(x) = (3x^2 - 2x)(x^2 - x + 2) + (x^3 - x + 1)(4x^3 - 1)$$

$$\rightarrow f'(1) = (3-2)(1-1+2) + (1-1+1)(4-1)$$

$$= (1)(2) + (1)(3) = \underline{\underline{5}}$$

5. Find the equation of the tangent line to the curve $y = \frac{x}{x+1}$ at $x = 1$.

(A) $y = \frac{1}{2}$ (B) $y = -\frac{1}{2}x + 1$ (C) $y = \frac{1}{2}x$

(D) $y = -\frac{1}{4}x + \frac{3}{4}$ $\boxed{\text{(E) } y = \frac{1}{4}x + \frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1(x+1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2} \rightarrow \text{at } x=1,\ m = y' = \frac{1}{2^2} = \frac{1}{4}$$

Point: $y(1) = \frac{1}{1+1} = \frac{1}{2} \rightarrow \left(1, \frac{1}{2}\right) = (x_0, y_0)$

$$y - y_0 = m(x - x_0) \rightarrow y - \frac{1}{2} = \frac{1}{4}(x-1)$$

$$\rightarrow y = \frac{1}{4}x - \frac{1}{4} + \frac{1}{2} = \underline{\underline{\frac{1}{4}x + \frac{1}{4}}}$$

6. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

(A) 1 $\boxed{\text{(B) } -1}$ (C) 0

(D) 1/2 (E) $\sqrt{2}/2$

$$f^{(4)}(x) = f^{(8)}(x) = \cdots = f^{(124)}(x) = \sin x, \text{ so}$$

$$f^{(125)}(x) = \frac{d}{dx}\left[f^{(124)}(x)\right] = \frac{d}{dx}(\sin x) = \cos x.$$

$$f^{(125)}(\pi) = \cos \pi = \underline{\underline{-1}}$$

7. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $g(x)$?

(A) x (B) x^2 (C) $\sin x$

(D) $\sin^2 x$ (E) None of the above

$\sin^2 x = (\sin x)^2$, so $g(x) = \sin x$,
$f(x) = x^2$

8. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\frac{dy}{dx}$ when $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

(A) 9 (B) 12 (C) 13

(D) 15 (E) 23

$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$

at $x=2$: $\frac{dy}{dx} = f'(2)g(2) + f(2)g'(2)$
$= (4)(1) + (3)(3)$
$= 4 + 9 = \underline{13}$

9. If $g(x) = \dfrac{ax+b}{cx+d}$, then $g'(1)$ is which of the following? Note: The numbers a, b, c, and d are constants.

(A) $\dfrac{a+b-c-d}{c+d}$ $\boxed{\text{(B) } \dfrac{ad-bc}{(c+d)^2}}$ (C) $\dfrac{a+b-c-d}{(c+d)^2}$

(D) $\dfrac{ad+bc}{c+d}$ (E) $\dfrac{ad+bc}{(c+d)^2}$

$$g'(x) = \frac{a(cx+d) - (ax+b)c}{(cx+d)^2}$$

$$= \frac{acx + ad - (acx + bc)}{(cx+d)^2}$$

$$= \frac{ad-bc}{(cx+d)^2} \rightarrow g'(1) = \underline{\underline{\frac{ad-bc}{(c+d)^2}}}$$

10. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\dfrac{3\pi}{4}$ $\boxed{\text{(B) } \dfrac{3\pi}{4} + \dfrac{1}{2}}$ (C) $\dfrac{1}{2}$

(D) $\dfrac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

$$f'(x) = 3x^2 \arctan(x) + x^3 \cdot \frac{1}{1+x^2}$$

$$\rightarrow f'(1) = 3(1)^2 \arctan(1) + 1^3 \cdot \frac{1}{1+1^2} = \underline{\underline{3\left(\frac{\pi}{4}\right) + \frac{1}{2}}}$$

11. Consider the functions $f(x) = \sin(x^2)$ and $g(x) = \sin^2(x)$. Which of the following is true?

~~(A)~~ $f'(x) = \cos(x^2)$ ~~(B)~~ $g'(x) = -2\sin(x)\cos(x)$ ~~(C)~~ $f'(x) = g'(x)$

~~(D)~~ $f'(\pi) = g'(\pi) = 0$ $\boxed{\text{(E) } f'(0) = g'(0)}$

$$f'(x) = 2x\cos(x^2) \neq \cos(x^2) \rightarrow \cancel{A}$$

$$f'(\pi) = 2\pi\cos\pi^2 \neq 0 \rightarrow \cancel{D}$$

$$g'(x) = 2\sin x\cos x \neq -2\sin x\cos x \rightarrow \cancel{B}$$

not equal $\rightarrow \cancel{C}$

$$f'(0) = 2(0)\cos(0^2) = 0 \leftarrow E \checkmark$$

$$g'(0) = 2\sin(0)\cos(0) = 2(0)(1) = 0$$

12. If $\frac{d}{dx}[f(4x)] = x^2$, then find $f'(x)$.

$\boxed{\text{(A) } \frac{x^2}{64}}$ (B) $\frac{x^2}{16}$ (C) $\frac{x^2}{4}$

(D) x^2 (E) $4x^2$

$$\frac{d}{dx}[f(4x)] = f'(4x)\cdot 4 = x^2 \rightarrow f'(4x) = \frac{x^2}{4}$$

Let $u = 4x$. Then $\frac{u}{4} = x$, so

$$f'(u) = \frac{\left(\frac{u}{4}\right)^2}{4} = \frac{\frac{u^2}{16}}{4} = \frac{u^2}{64}.$$

replacing u with x to represent the function,

$$\underline{f'(x) = \frac{x^2}{64}}$$

13. Find an equation of the tangent line to the curve $(x^2+y^2)^2 = 4x^2y$ at the point $(1,1)$.

$\boxed{\text{(A) } y = 1}$ (B) $y = x$ (C) $y = 2x - 1$

(D) $y = -x + 2$ (E) $y = -2x + 3$

$$\frac{d}{dx}\left[(x^2+y^2)^2\right] = \frac{d}{dx}\left[4x^2y\right]$$

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = 8xy + 4x\frac{dy}{dx}$$

at $x=1, y=1$:

$$2(1+1)\left(2+2\frac{dy}{dx}\right) = 8 + 4\frac{dy}{dx}$$

$$4\left(2+2\frac{dy}{dx}\right) = 8+4\frac{dy}{dx}$$

$$8+8\frac{dy}{dx} = 8+4\frac{dy}{dx} \rightarrow 4\frac{dy}{dx} = 0 \rightarrow \frac{dy}{dx} = 0$$

point $(1,1)$, slope $m=0$

$y-1 = 0(x-1)$

$y = 1$

14. Find $\frac{d}{dx}\left[\sin(\ln x^2)\right]$.

(A) $\frac{-\cos(\ln(x))}{x^2}$ (B) $\frac{-2\sin(\ln(x^2))}{x^2}$ (C) $\frac{\cos(\ln(x))}{2x^2}$

$\boxed{\text{(D) } \frac{2\cos(\ln(x^2))}{x}}$ (E) None of the above

$$\frac{d}{dx}\left[\sin(\ln x^2)\right] = \cos(\ln x^2)\cdot\frac{d}{dx}\left[\ln(x^2)\right]$$

$$= \cos(\ln x^2)\cdot\frac{1}{x^2}\cdot\frac{d}{dx}(x^2)$$

$$= \cos(\ln x^2)\cdot\frac{1}{x^2}\cdot 2x$$

$$= \frac{2x\cos(\ln x^2)}{x^2} = \frac{2\cos(\ln(x^2))}{x}$$

15. Find $\frac{d}{dx}[\log_4(3x)]$.

(A) $\frac{1}{3x \ln 4}$ $\boxed{\text{(B) } \frac{1}{x \ln 4}}$ (C) $\frac{1}{x}$

(D) $\frac{3}{x \ln 4}$ (E) $\frac{3}{x}$

$$\frac{d}{dx}\left[\log_4(3x)\right] = \frac{1}{3x\ln 4}\frac{d}{dx}(3x) = \frac{3}{3x\ln 4} = \underline{\underline{\frac{1}{x\ln 4}}}$$

16. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

✓ I. $k > 0$

~~II.~~ $P'(5) < 0$

✓ III. $P'(10) = 100ke^{10k}$

$\boxed{\text{(A) I and III only.}}$ (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

I. P increasing, so $k > 0$ in $P = P(0)e^{kt}$ ✓

II. $P'(t) = kP(t) > 0$ since $k > 0$ and $P > 0$
$\rightarrow P'(5) > 0$ ✗

III. $P'(t) = 100ke^{kt} \rightarrow P'(10) = 100ke^{10k}$ ✓

17. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

$\boxed{\text{(D) } 10e^{-t\ln(2)/20}}$ (E) $10e^{t\ln(2)/20}$

$y = Ae^{kt}$, $A = y(0) = 10$: $y = 10e^{kt}$

At $t=20$, $y=5$: $5 = 10e^{k(20)}$

$\to \frac{1}{2} = e^{20k} \to 20k = \ln\frac{1}{2}$

$k = \frac{\ln\frac{1}{2}}{20}$

$= \frac{-\ln 2}{20}$

$\to y = 10e^{kt} = 10e^{-t(\ln 2)/20}$

18. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by,

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ $\boxed{\text{(E) } 1013e^{h\ln(0.88)/1000}}$

$y = Ae^{kh}$, $A = y(0)$ = pressure at $h=0$ (sea level)

$= 1013 \to y = 1013e^{kh}$

At $h=1000$, $y = 0.88 \cdot 1013$ = 88% of sea level pressure

$\to 0.88\cancel{(1013)} = \cancel{1013}e^{k(1000)}$

$0.88 = e^{1000k} \to 1000k = \ln(.88)$

$\to k = \frac{\ln(.88)}{1000}$

So $y = 1013e^{h\ln(0.88)/1000}$

19. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

$y = \sqrt[3]{x^4+11} \rightarrow y^3 = x^4 + 11$

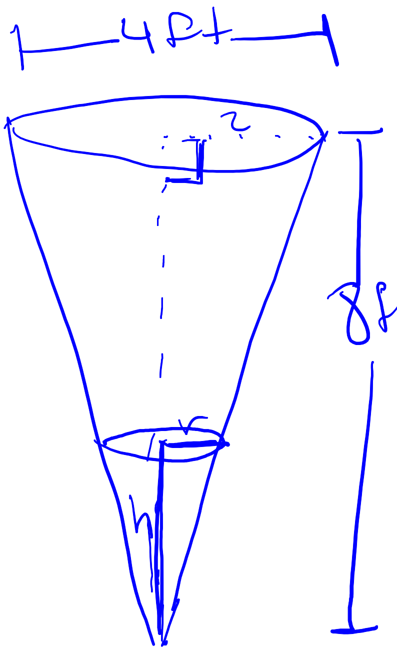
$\frac{d}{dt}(y^3) = \frac{d}{dt}(x^4+11) \rightarrow 3y^2 \frac{dy}{dt} = 4x^3 \frac{dx}{dt}$

When $x = 2$, $y = 3$, and $\frac{dy}{dt} = 32$:

$3(3)^2(32) = 4(2)^3 \frac{dx}{dt} \rightarrow 27 \cdot \cancel{32} = \cancel{32} \frac{dx}{dt}$

$\frac{dx}{dt} = 27$ cm/s

20. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3} r^2 h$?)



(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min

similar triangles: $\frac{2}{r} = \frac{8}{h} \rightarrow 2h = 8r$

$\rightarrow r = \frac{h}{4}$

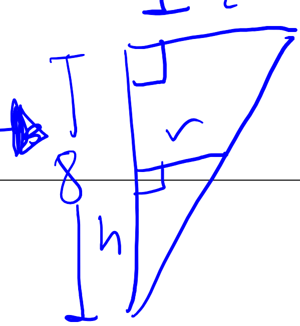
Plug $r = \frac{h}{4}$ into Vol. formula:

$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h = \frac{\pi}{48} h^3$

$\frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \frac{dh}{dt} = \frac{\pi h^2}{16} \frac{dh}{dt}$

when $\frac{dV}{dt} = -2$ and $h = 2$,

$-2 = \frac{\pi(2^2)}{16} \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{-32}{4\pi} = -\frac{8}{\pi}$



$\rightarrow$ falling @ rate of $\frac{8}{\pi}$ ft/min

21. Determine $f''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

(A) $\frac{-1}{2x^2}$ (B) $\frac{6\ln x}{x^4}$ (C) $\frac{1-6\ln x}{x^4}$

(D) $\frac{1-2\ln x}{x^3}$ $\boxed{\text{(E) None of the above}}$

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - (\ln x)(2x)}{x^4} = \frac{x - 2x\ln x}{x^4}$$

$$f''(x) = \frac{\left[1 - \left(2\ln x + 2x \cdot \frac{1}{x}\right)\right](x^4) - 4x^3\left[x - 2x\ln x\right]}{x^8}$$

$$= \frac{x^4\left[1 - 2\ln x - 2 - 4(1 - 2\ln x)\right]}{x^8} = \frac{-1-4-2\ln x+8\ln x}{x^4}$$

$$= \frac{-5+6\ln x}{x^4}$$

22. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ $\boxed{\text{(C) } \frac{17}{8}}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

$$L(x) = f'(1)(x-1) + f(1)$$

$$f(1) = \sqrt{1+2+1} = \sqrt{4} = 2$$

$$f'(x) = \frac{1}{2}(x^3+2x+1)^{-1/2}(3x^2+2)$$

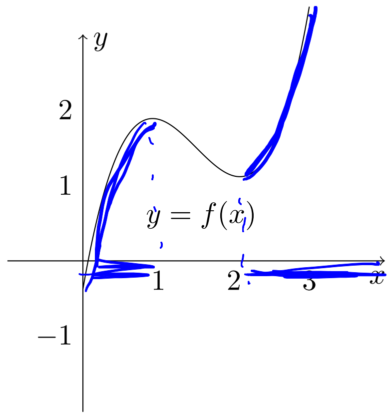
$$f'(1) = \frac{1}{2}(1+2+1)^{-1/2}(3+2) = \frac{1}{2}\left(\frac{1}{2}\right)(5) = \frac{5}{4}$$

$$\rightarrow f(1.1) \approx L(1.1) = f'(1)(1.1-1) + f(1) = \frac{5}{4}(0.1) + 2$$

$$= \frac{5}{40} + 2 = \frac{1}{8} + 2$$

$$= \underline{\underline{\frac{17}{8}}}$$

23. The curve below is the graph of $y = f(x)$. List all x-values, in interval form, on which $f'(x)$ (the *derivative* of f) is positive. [1]



(A) $(0, 1)$ (B) $(0, 2)$ (C) $(1, 2)$

(D) $(2, 3)$ (E) $(0, 1)$ and $(2, 3)$

f'(x) is positive when f is increasing, between about x=0 and x=1, and between about x=2 and x=3.