

Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

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**Solutions should show all of your work, not just a single final answer.**

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## 4.3: How Derivatives Affect the Shape of a Graph

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1. For the following functions, (i) determine all open intervals where  $f(x)$  is increasing, decreasing, concave up, and concave down, and (ii) find all local maxima, local minima, and inflection points. Give all answers **exactly**, not as numerical approximations.

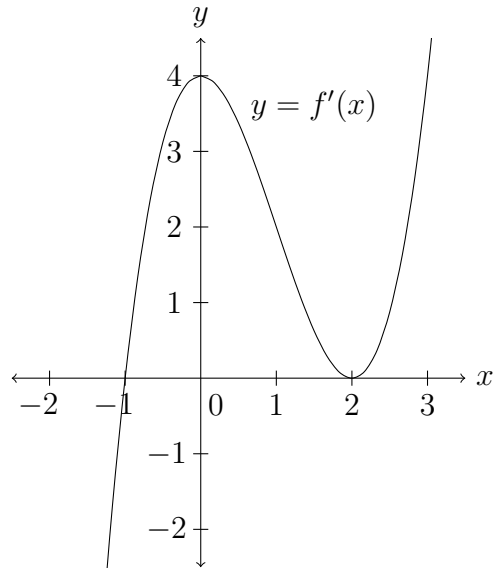
(a)  $f(x) = x^5 - 2x^3$  for all  $x$

(b)  $f(x) = x - 2 \sin x$  for  $-2\pi < x < 2\pi$

(c)  $f(x) = e^{-x} - e^{-3x}$  for  $x > 0$

2. For  $x$  in the interval  $(0, 100)$ , let  $f(x) = x^{100} + (100 - x)^{100}$ . Determine on what open intervals in  $(0, 100)$  the function  $f(x)$  is increasing and decreasing, and use this information to decide which of  $33^{100} + 67^{100}$  or  $41^{100} + 59^{100}$  is larger.

3. Below is a graph of  $y = f'(x)$  for some function  $f(x)$ . Determine the intervals where  $f(x)$  is increasing and decreasing, the  $x$ -values where  $f(x)$  has local maxima and minima, and the  $x$ -values where  $f(x)$  has inflection points.



4. T/F (with justification) If a function  $f(x)$  on the interval  $(-1, 1)$  is twice differentiable and  $f''(c) = 0$  for some  $c$  in  $(-1, 1)$  then  $f(x)$  has an inflection point at  $x = c$ .

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## 4.4: Indeterminate Forms and l'Hospital's Rule

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5. For each of the following limits, indicate what kind of indeterminate form it is and then evaluate it with l'Hospital's rule.

(a)  $\lim_{x \rightarrow 0} \frac{(x+1)^{11} - 11x - 1}{x^2}$

(b)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{e^{9x} - e^{2x}}$

(c)  $\lim_{x \rightarrow 0} \frac{x - \tan x}{x - \sin x}$

$$(d) \lim_{x \rightarrow \infty} \frac{\ln(1881x^2 + 1)}{\ln x}$$

$$(e) \lim_{x \rightarrow 0} \frac{\ln(\cos(2x))}{\ln(\cos(3x))}$$

6. Indicate what kind of indeterminate form  $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$  is and then try to evaluate it with l'Hospital's rule. Explain what goes wrong and then evaluate this limit using methods from earlier in the course.