Math 1131 Week 2 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

2.3: Calculating Limits Using the Limit Laws

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1. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

(a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.

(b) Evaluate the following limits if they exist. (If a limit does not exist, write DNE.)

(i) $\lim_{x \to 1^-} f(x)$

(ii) $\lim_{x \to 1^+} f(x)$

(iii) $\lim_{x \to 1} f(x)$

(iv) $\lim_{x \to 2^-} f(x)$

(v) $\lim_{x \to 2^+} f(x)$

(vi) $\lim_{x \to 2} f(x)$

2. Evaluate the following limits exactly using algebra and limit laws (some limits may be DNE).

(a) $\lim_{x \to 2} \frac{x^3 - 2}{2x^2 - 3x + 2}$

(b) $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

(c) $\lim_{x \to 3} \frac{\sqrt{x^2 + 40} - 7}{x - 3}$

(d) $\lim_{x \to -2} \sqrt{x^4 + 3x + 6}$

(e) $\lim_{x \to 1} \frac{x^2 + 4x}{x^2 + 3x - 4}$

(f) $\lim_{x \to 1} \frac{(x^2 + x)^2 - 4}{x^2 + x - 2}$

3. Evaluate the following limits using algebra and limit laws (some limits may be DNE). Note that a represents a constant, and answers may be in terms of a.

   (a) $\lim_{t\to 0} \dfrac{\sqrt{a+t}-\sqrt{a-t}}{t}$ for $a > 0$

   (b) $\lim_{h\to 0} \dfrac{1/(a+h)^2 - 1/a^2}{h}$ for $a \neq 0$

4. T/F (with justification) If $\lim_{x\to 2} g(x) = 0$ and $\lim_{x\to 2} h(x) = 0$ then $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$ does not exist.

2.5: Continuity

5. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

(a) Determine the value of a for which $f(x)$ is continuous from the left at 1.

(b) Determine the value of a for which $f(x)$ is continuous from the right at 1.

(c) Is there a value of a for which $f(x)$ is continuous at 1? Explain.

6. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $(2, 3)$. Clearly explain your reasoning.

7. Let

$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x > 1 \end{cases}$$

with the value of $f(1)$ to be determined.

(a) Compute $\lim_{x \to 1^-} f(x)$ in terms of k.

(b) Compute $\lim_{x \to 1^+} f(x)$ in terms of k.

(c) Find the values of k and $f(1)$ that make $f(x)$ continuous at $x = 1$.

(d) Using the choice of k and $f(1)$ in part (c), make a graph of $y = f(x)$ for $0 \leq x \leq 2$.

8. The function $f(x)$ is continous on the interval $(-3, 4)$. If we know that $f(-1) = 4$ and $f(3) = 7$, what can we say about the outputs of $f(x)$, i.e. what values does f definitely take and/or not take?

9. T/F (with justification) The function

$$f(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ 1 + \cos x & \text{if } x > 0 \end{cases}$$

has a jump discontinuity at $x = 0$.

10. T/F (with justification) A function that is continuous at a point has to be defined at the point.

11. T/F (with justification) A function that is discontinuous at a point can't be defined at the point.

2.6: Limits at Infinity and Horizontal Asymptotes

12. Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

    (a) $\displaystyle\lim_{x\to\infty} \frac{2x+3}{6x-7}$

    (b) $\displaystyle\lim_{x\to-\infty} \frac{x^3}{\sqrt{6x^4-1}}$

    (c) $\displaystyle\lim_{x\to\infty} \sqrt{x^2+3x}-x$

    (d) $\displaystyle\lim_{x\to\infty} \frac{100000x}{x^3+x}$

(e) $\lim_{x\to\infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$

(f) $\lim_{x\to-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(g) $\lim_{x\to\infty} \sqrt{x} + \sin x$

(h) $\lim_{x\to\infty} \frac{1}{x} + \sin x$

13. Let $f(x) = \dfrac{\sqrt{4x^6+5}}{x^3-1}$.

   (a) Compute $\lim_{x\to\infty} f(x)$.

   (b) Compute $\lim_{x\to-\infty} f(x)$.

   (c) What are the horizontal asymptotes of the graph of $y = f(x)$?

   (d) What is the vertical asymptote of the graph of $y = f(x)$?

14. T/F (with justification) The graph of the function $y(x) = 3 + 6e^{-kx}$, with k a positive constant, has a horizontal asymptote $y = 6$.

15. T/F (with justification) If the continuous function $f(x)$ has domain $(-\infty, +\infty)$, then either $\lim_{x\to\infty} f(x)$ exists or $\lim_{x\to\infty} f(x)$ is $\pm\infty$.