University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 2

Sections Covered: 3.6, 3.8, 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.7, 4.8

Read This First!

- The exam will be 50 minutes, timed, and administered via HuskyCT.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, please carefully check all submitted answers. The submitted letter answers are the **ONLY** place that counts as your official answers.
- You may use a calculator on the exam. No books or other references or are permitted, and **you are expected to work independently.**

1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

   (A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ (B) $x_{n+1} = (x_n^2 + 7)/(2x_n)$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

   (D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

2. Find $\frac{d}{dx}\left[\sin(\ln x^2)\right]$.

   (A) $\frac{-\cos(\ln(x))}{x^2}$ (B) $\frac{-2\sin(\ln(x^2))}{x^2}$ (C) $\frac{\cos(\ln(x))}{2x^2}$

   (D) $\frac{2\cos(\ln(x^2))}{x}$ (E) None of the above

3. Find $\frac{d}{dx}[\log_4(3x)]$.

   (A) $\frac{1}{3x\ln 4}$ (B) $\frac{1}{x\ln 4}$ (C) $\frac{1}{x}$

   (D) $\frac{3}{x\ln 4}$ (E) $\frac{3}{x}$

4. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

   I. $k > 0$

   II. $P'(5) < 0$

   III. $P'(10) = 100ke^{10k}$

   (A) I and III only. (B) I and II only. (C) I only.

   (D) II only. (E) I, II, and III.

5. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

   (A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

   (D) $10e^{-t\ln(2)/20}$ (E) $10e^{t\ln(2)/20}$

6. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by,

   (A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

   (D) $1000e^{-h\ln(2)/20}$ (E) $1013e^{h\ln(0.88)/1000}$

7. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

8. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3}r^2h$?)

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min

9. Determine $f''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

(A) $\frac{-1}{2x^2}$ (B) $\frac{6\ln x}{x^4}$ (C) $\frac{1 - 6\ln x}{x^4}$

(D) $\frac{1 - 2\ln x}{x^3}$ (E) None of the above

10. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ (C) $\frac{17}{8}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

11. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 (B) 19/6 (C) 15/4

(D) 12/7 (E) 17/6

12. Which of the following is the absolute maximum value of the function $f(x) = \frac{x}{x^2 + 4}$ on the interval $[0, 4]$?

(A) $\frac{1}{8}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$

(D) $\frac{1}{2}$ (E) 1

13. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

(A) 9 (B) $\sqrt{27}$ (C) $\sqrt{3}$

(D) 3 (E) No such value of c exists.

14. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

(A) 1 (B) -2 (C) $-2,\ 1$

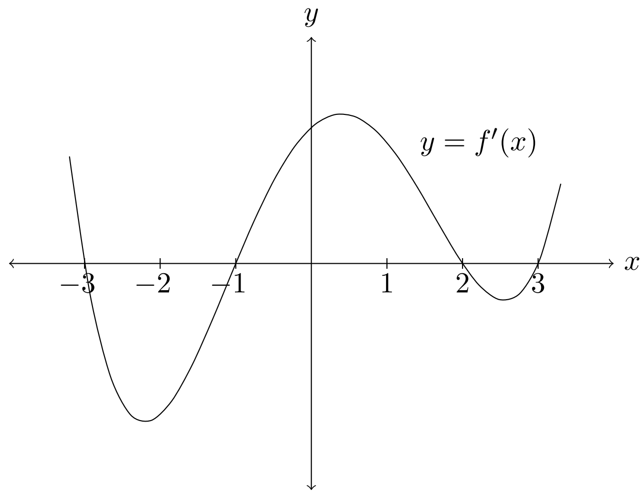
(D) $-2,\ \frac{1}{2}$ (E) $-2,\ \frac{1}{2},\ 1$

15. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

16. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



(A) Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

17. Referring to the same graph of the derivative in question 16, at approximately what x-value(s) is $f(x)$ concave up?

(A) $x < -1$ and $x > 1.5$

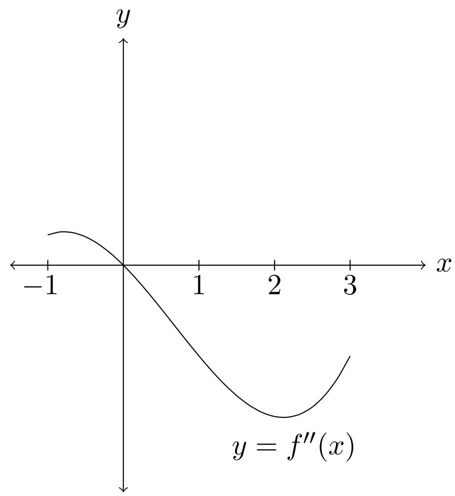
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

18. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$.

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$.

(C) The function $f(x)$ has a point of inflection at $x = 0$.

(D) The derivative $f'(x)$ has a local maximum at $x = 0$.

(E) All of the above.

19. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

20. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}.$$

(A) $+\infty$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

21. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}.$$

(A) 0 (B) 1 (C) $+\infty$

(D) -1 (E) 1/2

22. Determine the number of inflection points of the graph of $y = x^2 - \dfrac{1}{x}$ on its domain.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) $24,\ 32$ (B) $26,\ 28$ (C) $20,\ 40$

(D) $26,\ 27$ (E) None of the above

24. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?

(A) \$500 (B) \$600 (C) \$1000

(D) \$1200 (E) \$2000

25. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x\to\infty} \frac{f(x)}{x^2}$$

(A) $\sin(x)$ (B) e^{-x} (C) $\cos(x)$

(D) $\ln(x)$ (E) All of the above

26. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \leq t \leq 2$?

(A) $1 - \ln 2$ (B) 0 (C) $2 - \ln 5$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

27. If $f(1) = 9$ and $f'(x) \geq 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

(A) 19 (B) 18 (C) 12

(D) Cannot be determined (E) None of the above

28. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6