



University of Connecticut
Department of Mathematics

MATH 1131

PRACTICE PROBLEMS FOR EXAM 2

Sections Covered: 3.6, 3.8, 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.7, 4.8

Read This First!

- The exam will be 50 minutes, timed, and administered via HuskyCT.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, please carefully check all submitted answers. The submitted letter answers are the **ONLY** place that counts as your official answers.
- You may use a calculator on the exam. No books or other references or are permitted, and **you are expected to work independently.**

1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

- (A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ (B) $x_{n+1} = (x_n^2 + 7)/(2x_n)$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$
 (D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$$\left. \begin{array}{l} x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \\ f(x_n) = x_n^2 - 7 \\ f'(x_n) = 2x_n \end{array} \right\} \rightarrow x_{n+1} = x_n - \frac{x_n^2 - 7}{2x_n}$$

$$= \frac{2x_n^2 - (x_n^2 - 7)}{2x_n}$$

$$= \frac{x_n^2 + 7}{2x_n}$$

2. Find $\frac{d}{dx} [\sin(\ln x^2)]$.

- (A) $\frac{-\cos(\ln(x))}{x^2}$ (B) $\frac{-2\sin(\ln(x^2))}{x^2}$ (C) $\frac{\cos(\ln(x))}{2x^2}$
 (D) $\frac{2\cos(\ln(x^2))}{x}$ (E) None of the above

$$\begin{aligned} \frac{d}{dx} [\sin(\ln x^2)] &= \cos(\ln x^2) \cdot \frac{d}{dx} [\ln(x^2)] \\ &= \cos(\ln x^2) \cdot \frac{1}{x^2} \cdot \frac{d}{dx} (x^2) \\ &= \cos(\ln x^2) \cdot \frac{1}{x^2} (2x) \\ &= \frac{2x \cos(\ln x^2)}{x^2} = \frac{2\cos(\ln x^2)}{x} \end{aligned}$$

3. Find $\frac{d}{dx} [\log_4(3x)]$.

- (A) $\frac{1}{3x \ln 4}$ (B) $\frac{1}{x \ln 4}$ (C) $\frac{1}{x}$
 (D) $\frac{3}{x \ln 4}$ (E) $\frac{3}{x}$

$$\frac{d}{dx} [\log_4(3x)] = \frac{1}{3x \ln 4} \frac{d}{dx} (3x) = \frac{3}{3x \ln 4} = \frac{1}{x \ln 4}$$

4. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

✓ I. $k > 0$

~~II.~~ $P'(5) < 0$

✓ III. $P'(10) = 100ke^{10k}$

- (A) I and III only. (B) I and II only. (C) I only.
(D) II only. (E) I, II, and III.

I. P increasing, so $k > 0$ ✓
 II. $P'(t) = kP(t) > 0$ since $k > 0$ and $P > 0 \rightarrow P'(5) > 0$ X
 III. $P'(t) = 100ke^{kt} \rightarrow P'(10) = 100ke^{10k}$ ✓

5. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

(D) $10e^{-t \ln(2)/20}$ (E) $10e^{t \ln(2)/20}$
 $y = Ae^{kt}$, $A = 10$. At $t = 20$, $y = 5$: $5 = 10e^{k(20)}$
 $\rightarrow \frac{1}{2} = e^{20k} \rightarrow 20k = \ln \frac{1}{2}$
 $\rightarrow k = \frac{\ln \frac{1}{2}}{20}$
 $k = -\frac{\ln 2}{20}$
 $y = 10e^{-(\ln 2)t/20}$

6. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by.

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h \ln(2)/20}$ (E) $1013e^{h \ln(0.88)/1000}$
 $y = Ae^{kh}$, $A = 1013$
 At $t = 1000$, $y = 0.88A$
 $\rightarrow 0.88(1013) = 1013e^{k(1000)}$
 $\rightarrow 0.88 = e^{1000k}$
 $\rightarrow \ln(0.88) = 1000k \rightarrow k = \frac{\ln(0.88)}{1000}$
 $\rightarrow y = 1013e^{\ln(0.88)h/1000}$

7. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point (2, 3), the y -coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x -coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

$$y^3 = x^4 + 11$$

$$\Rightarrow 3y^2 \frac{dy}{dt} = 4x^3 \frac{dx}{dt}$$

When $x=2, y=3, \frac{dy}{dt}=32$:

$$3(3)^2 (32) = 4(2)^3 \frac{dx}{dt}$$

$$27 \cdot 32 = 32 \frac{dx}{dt}$$

$$\frac{dx}{dt} = 27 \text{ cm/s}$$

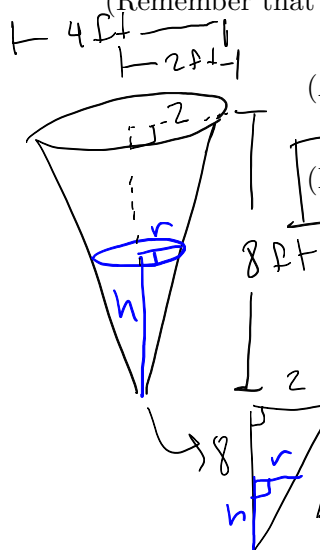
8. Water is withdrawn at a constant rate of $2 \frac{\text{ft}^3}{\text{min}}$ from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft?

(Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3} r^2 h$?)

$$h=2$$

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min



$$V = \frac{\pi}{3} \left(\frac{h}{4}\right)^2 h = \frac{\pi}{3} \cdot \frac{h^3}{16} = \frac{\pi}{48} h^3 = V$$

$$\Rightarrow \frac{dV}{dt} = \frac{3\pi}{48} h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = -2, h=2: -2 = \frac{\pi}{16} (2)^2 \frac{dh}{dt}$$

$$\Rightarrow -2 = \frac{\pi}{4} \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \underline{\underline{-\frac{8}{\pi}}}$$

9. Determine $f'''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

(A) $\frac{-1}{2x^2}$ (B) $\frac{6 \ln x}{x^4}$ (C) $\frac{1 - 6 \ln x}{x^4}$

(D) $\frac{1 - 2 \ln x}{x^3}$ (E) None of the above

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - (\ln x)(2x)}{x^4} = \frac{x - 2x \ln x}{x^4}$$

$$f''(x) = \frac{[1 - (2 \ln x + 2x \cdot \frac{1}{x})](x^4) - 4x^3 [x - 2x \ln x]}{x^8} = \frac{-5 + 6 \ln x}{x^4}$$

$$= \frac{x^4 [1 - 2 \ln x - 2 - 4(1 - 2 \ln x)]}{x^8} = \frac{-1 - 4 - 2 \ln x + 8 \ln x}{x^4}$$

10. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$

(B) $\frac{21}{10}$

(C) $\frac{17}{8}$

(D) $\frac{1}{2}$

(E) $\frac{17}{16}$

$$L(x) = f'(1)(x-1) + f(1)$$

$$f(1) = \sqrt{1+2+1} = 2$$

$$f'(x) = \frac{1}{2}(x^3 + 2x + 1)^{-1/2}(3x^2 + 2)$$

$$f'(1) = \frac{1}{2}(1+2+1)^{-1/2}(3+2)$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)(5) = \frac{5}{4}$$

$$\rightarrow f(1.1) \approx L(1.1) = f'(1)(1.1-1) + f(1) = \frac{5}{4}(0.1) + 2$$

$$= \frac{1}{8} + \frac{16}{8} = \frac{17}{8}$$

11. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2 .

(A) $1/2$

(B) $19/6$

(C) $15/4$

(D) $12/7$

(E) $17/6$

$$f'(x) = 2x \rightarrow f'(3) = 6$$

$$f(3) = 9 - 10 = -1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-1}{6} = \frac{18}{6} + \frac{1}{6} = \frac{19}{6}$$

12. Which of the following is the absolute maximum value of the function $f(x) = \frac{x}{x^2 + 4}$ on the interval $[0, 4]$?

(A) $\frac{1}{8}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) 1

$$f'(x) = \frac{(x^2+4) \cdot 1 - x(2x)}{(x^2+4)^2}$$

$$= \frac{-x^2+4}{(x^2+4)^2}$$

Crit #s: $x^2 = 4 \rightarrow x = \pm 2$
(denom never 0)

Only one crit # in $[0, 4]$: $x = 2$

so $f(0) = 0$
 $f(2) = \frac{2}{4+4} = \frac{1}{4}$ \rightarrow (largest)
 $f(4) = \frac{4}{16+4} = \frac{1}{5}$

13. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

(A) 9 (B) $\sqrt{27}$ (C) $\sqrt{3}$
(D) 3 (E) No such value of c exists.

1) f cont. on $[0, 3]$ ✓
2) f diff'ble on $(0, 3)$ ✓
so MVT applies ✓

$$f'(c) = \frac{f(3) - f(0)}{3 - 0}$$

$$3c^2 = \frac{3^3 - 0^3}{3 - 0} \rightarrow 3c^2 = \frac{27}{3} = 9 \rightarrow c^2 = 3 \rightarrow c = \pm\sqrt{3} \rightarrow c = \sqrt{3}$$

($c = -\sqrt{3}$ not in $(0, 3)$)

14. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

(A) 1 (B) -2 (C) -2, 1
(D) -2, $\frac{1}{2}$ (E) -2, $\frac{1}{2}$, 1

$$f'(x) = 6x^2 + 6x - 12$$

$$= 6(x^2 + x - 2)$$

$$= 6(x - 1)(x + 2) = 0$$

$$x = 1, -2 \text{ crit. \#s}$$

2nd deriv. test:

$$f''(x) = 12x + 6$$

$$f''(1) = 18 > 0$$

$$f''(-2) = -18 < 0 \rightarrow \text{loc. max}$$

15. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1

(C) 2

(D) 3 (E) 4

$$f'(x) = 4x^3 - 16x$$

$$f''(x) = 12x^2 - 16 = 0$$

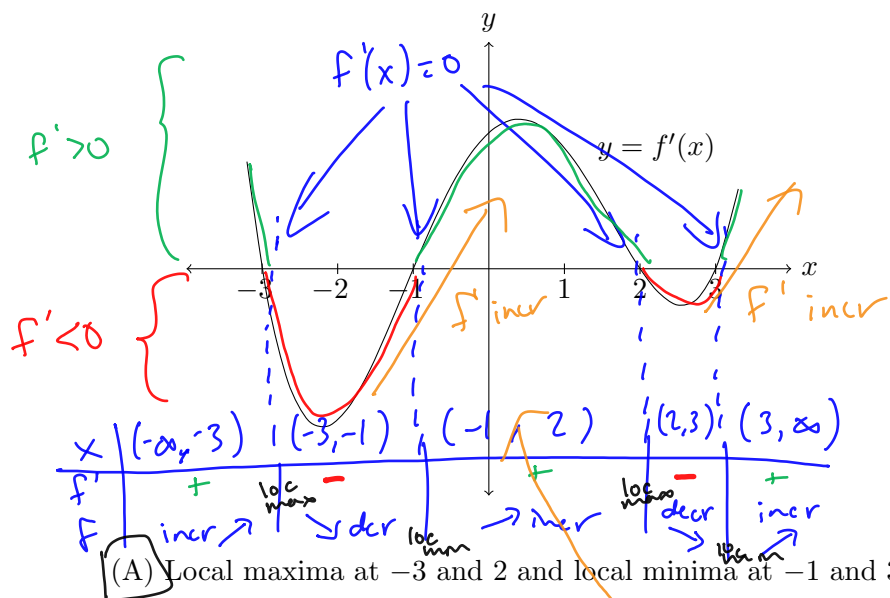
$$x^2 = \frac{16}{12} = \frac{4}{3}$$

$$x = \pm \frac{2}{\sqrt{3}}$$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	-	+
f	c.u.	c.d.	c.u.

inflection points (2)

16. Below is the graph of the derivative $f'(x)$ of a function $f(x)$. At what x -value(s) does $f(x)$ have a local maximum or local minimum?



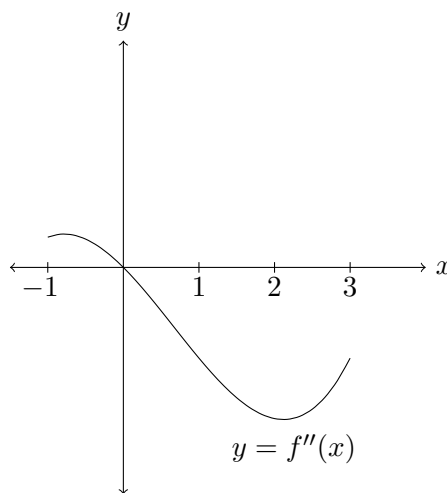
- (B) Local maxima at -1 and 3 and local minima at -3 and 2
- (C) Local maxima at -1 and 3 and local minimum at 2
- (D) Local maxima at -3 and 2 and local minimum at -1
- (E) None of the above

17. Referring to the same graph of the derivative in question 5, at approximately what x -value(s) is $f(x)$ concave up? $\Leftrightarrow f'' > 0 \Leftrightarrow f'$ increasing (2 intervals):

- (A) $x < -1$ and $x > 1.5$
- (B) $-1 < x < 2$
- (C) $-2.1 < x < .8$ and $x > 2.6$
- (D) $-\infty < x < \infty$
- (E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

$(-2.1, 0.8)$ and $(2.6, \infty)$

18. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



- (A) The function $f(x)$ is concave up when $-1 < x < 0$. $f'' > 0$ here ✓
- (B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. $f'' < 0$ here ✓
- (C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to - here ✓
- (D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to - here ✓
- ☒ (E) All of the above. $(f' \text{ changes } \nearrow \text{ to } \searrow \text{ here})$ ✓

19. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

- (A) $(-\infty, 1)$ only ☒ (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$
 (D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$$f'(x) = 4x^3 - 18x^2 + 24x$$

$$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$$

$x = 2, 1$

x	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
f''	+	-	+
f	CU	CD	CU

20. Evaluate the following limit:

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} \quad \text{"0/0" use L'Hospital's:}$$

(A) $+\infty$ (B) $-\infty$

(C) 0

(D) $1/2$ (E) $-1/2$

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x^2} \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\cos x}{2x} = \frac{1}{0^+} = +\infty$$

21. Evaluate the following limit:

$$\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{\cos x} \quad \text{"0/0" use L'Hospital's:}$$

(A) 0

(B) 1

(C) $+\infty$

(D) -1

(E) $1/2$

$$\lim_{x \rightarrow \pi/2} \frac{1 - \sin x}{\cos x} \stackrel{L'H}{=} \lim_{x \rightarrow \pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$$

22. Determine the number of inflection points of the graph of
- $y = x^2 - \frac{1}{x}$
- on its domain.

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$$y' = 2x + \frac{1}{x^2} \rightarrow y'' = 2 - \frac{2}{x^3}$$

$$y'' = 0 \text{ or DNE: } x = 0 \text{ or } 2 = \frac{2}{x^3} \rightarrow x^3 = 1 \rightarrow x = 1$$

x	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
y''	+	-	+
y	∪	∩	∪

inflection point at $x=1$ (not at $x=0$, a vertical asymptote)

23. Find two positive numbers
- x
- and
- y
- satisfying
- $y + 2x = 80$
- whose product is a maximum.

(A) 24, 32

(B) 26, 28

(C) 20, 40

maximize (D) 26, 27 (E) None of the above

$$A = xy \text{ with } y + 2x = 80: y = 80 - 2x, \text{ so}$$

$$A(x) = x(80 - 2x) = 80x - 2x^2 \rightarrow \text{maximize on } (0, 40)$$

$$A'(x) = 80 - 4x = 0 \text{ (or DNE)}$$

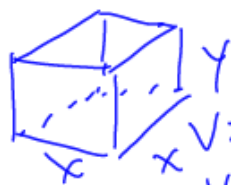
$$\rightarrow x = 20 \rightarrow y = 80 - 2x = 80 - 40 = 40$$

2nd deriv. test:

$$A''(x) = -4 < 0$$

so maximum ✓

24. A box with square base and open top must have a volume of 4000 cm^3 . If the cost of the material used is $\$1/\text{cm}^2$, then what is the smallest possible cost of the box?



(A) \$500 (B) \$600 (C) \$1000

(D) \$1200 (E) \$2000

$V = x^2 y$
 $V = 4000$
 $\rightarrow y = \frac{4000}{x^2}$

cost = $(\$1)(\text{area of bottom} + \text{sides})$
 $= x^2 + 4xy \rightarrow C(x) = x^2 + 4x \left(\frac{4000}{x^2} \right) = x^2 + \frac{16000}{x}$

$C'(x) = 2x - \frac{16000}{x^2} \Rightarrow 0 \text{ or DNE} \rightarrow x \neq 0, x = 20$

$C''(20) > 0$,
 so minimum ✓
 $\rightarrow C(20) = 1200$

minimize over $(0, \infty)$.

25. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$\lim_{x \rightarrow \infty} \sin x \rightarrow \text{DNE}$
 $\lim_{x \rightarrow \infty} e^{-x} = 0$
 $\lim_{x \rightarrow \infty} \frac{f(x)}{x^2} = \frac{?}{\infty}$ L'H applies if

(A) $\sin(x)$ (B) e^{-x} (C) $\cos(x)$
 (D) $\ln(x)$ (E) All of the above

$\lim_{x \rightarrow \infty} \ln x = \infty$ ✓
 $\lim_{x \rightarrow \infty} \cos x \rightarrow \text{DNE}$

$\lim_{x \rightarrow \infty} F(x) = \infty$

cont on [0, 2] so use closed interval method

26.

A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \leq t \leq 2$?

(A) $1 - \ln 2$

(B) 0

(C) $2 - \ln 5$

(D) $\ln 2 - 1$

(E) $\ln 5 - 2$

$$v'(t) = 1 - \frac{2t}{t^2 + 1} = 0, \text{ DNE} \rightarrow t^2 + 1 = 2t$$

$$\rightarrow t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

Crit # in [0, 2]

$$v(0) = 0 - \ln(1) = 0 \quad v(2) = 2 - \ln(5) \approx 0.39$$

$$v(1) = 1 - \ln(2) \approx 0.31$$

maximum

27.

If $f(1) = 9$ and $f'(x) \geq 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

using MVT: $f'(c) = \frac{f(4) - f(1)}{4 - 1}$ for some c in $[1, 4]$, so

(A) 19

(B) 18

(C) 12

(D) Cannot be determined

(E) None of the above

$$3f'(c) = f(4) - f(1)$$

$$\rightarrow f(4) = 3f'(c) + f(1)$$

$$= 3f'(c) + 9$$

$$\geq 3(3) + 9 = 18$$

=

28

Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0

crit #s: $x = -3, -2, 0, 4$
(where $f'(x) = 0$)

2nd deriv test:

$f'' > 0 \rightarrow \text{loc min}$

$f'' < 0 \rightarrow \text{loc max}$

$f' = 0 \rightarrow \text{CBD}$

min CBD min
 $x = -3, x = -2, x = 0$

max
 $x = 4$

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6