3.1: Derivatives of Polynomials and Exponential Functions

1. Use differentiation rules from Section 3.1 (not other methods) to compute the derivative of the following functions.

   (a) $f(x) = 7x^3 - 5x + 8$

   (b) $f(x) = e^x + xe^x$

   (c) $f(x) = 3x + \sqrt{3x}$

   (d) $f(x) = \sqrt{x} - 4e^x$

   (e) $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

   (f) $f(x) = \frac{12}{x^5} - \frac{7}{\sqrt{x}}$
2. Use differentiation rules to find the equation of the tangent line to $y = x^2 - x^4$ (see below) at the point $(1, 0)$.

3. Use differentiation rules to find the values of $a$ and $b$ that make the function

$$f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 2, \\
  ax^3 + bx & \text{if } x > 2 
\end{cases}$$

differentiable at $x = 2$. 
4. Find all points \((c, f(c))\) on the graph of \(f(x) = x^3 - 3x^2\) where the tangent line has slope 9.

5. T/F (with justification) If \(f(x) = \sqrt{7}\) for all \(x\), then \(f'(x) = \frac{1}{2\sqrt{7}}\) for all \(x\).
6. Compute the derivative of each function below using the methods from Sections 3.1 and 3.2 (not other methods).

(a) \( f(x) = \frac{x}{x+3} \) (simplify numerator in final answer)

(b) \( f(x) = \frac{e^x}{1+e^x} \) (simplify numerator in final answer)

(c) \( f(x) = \sqrt{x}e^x \)
(d) \( f(x) = \frac{e^x}{x^n} \) for constant \( n \), in two ways: (i) quotient rule and (ii) product rule

(e) \( f(x) = \frac{1}{x} + \frac{1}{1 - x} \) (in final answer, use a common denominator and simplify numerator)
7. In the function \( h(x) \) below, defined in terms of \( f(x) \) and \( g(x) \), determine \( h'(2) \) in each case if \( f(2) = 3 \), \( g(2) = 4 \), \( f'(2) = 1 \), and \( g'(2) = -5 \).

(a) \( h(x) = 2f(x) + 5g(x) \)

(b) \( h(x) = f(x)g(x) \)

(c) \( h(x) = \frac{f(x)}{g(x)} \)

(d) \( h(x) = \frac{g(x)}{f(x) + 2} \)
8. Compute the derivative of each function below using differentiation rules.

(a) \[ f(x) = x^3 \cos x \]

(b) \[ f(x) = \frac{1 + \sin x}{1 + \cos x} \]

(c) \[ f(x) = e^x \tan x \]

(d) \[ f(x) = \frac{\sec x}{\sqrt{x}} \]  (Compute (d) in two ways, using (i) the quotient rule and (ii) the product rule.)
9. Find the equation of the tangent line to the curve \( y = \sin x \cos x \) at \( x = \frac{\pi}{4} \). (Your coefficients must be exact, not approximations.)

10. Find the higher derivative \( \frac{d^{1881}}{dx^{1881}}(2 \cos x) \) by finding the first eight derivatives and observing the pattern that occurs.
11. Determine the following limits by making a change of variables to allow you to use the relation \( \lim_{t \to 0} \frac{\sin t}{t} = 1 \).

(a) \( \lim_{x \to 0} \frac{\sin 4x}{x} \)

(b) \( \lim_{x \to 0} \frac{\sin 7x}{5x} \)
Answers to selected problems

1. (a) \( f'(x) = 21x^2 - 5 \)
   
   (b) \( f'(x) = e^x + xe^{-1} \)

   (c) \( f'(x) = 3 + \frac{\sqrt{3}}{2\sqrt{x}} \)

   (d) \( f'(x) = \frac{1}{4x^{3/4}} - 4e^x \).

   (e) \( f'(x) = \frac{3}{2\sqrt{x}} + \frac{2}{\sqrt{x}} - \frac{3}{2x^{3/2}} \).

   (f) \( f'(x) = -\frac{60}{x^6} + \frac{7}{5x^{6/5}} \).

2. \( y = -2x + 2 \)

3. \( a = 1/4, b = 1 \).

4. \((-1, f(-1)) = (-1, -4) \) and \((3, f(3)) = (3, 0) \).

5. False

6. (a) \( \frac{3}{(x+3)^2} \)

   (b) \( \frac{e^x}{(1+e^x)^2} \)

   (c) \( \frac{(2x+1)e^x}{2\sqrt{x}} \)

   (d) (i): \( \frac{x^n e^x - e^x nx^{n-1}}{x^{2n}} \)

      (ii): \( e^x x^{-n} - ne^x x^{-n-1} \) (show why these are the same!)

   (e) \( \frac{2x-1}{x^2(1-x)^2} \)

7. (a) \(-23 \)

   (b) \(-11 \)

   (c) \( \frac{19}{16} \)

   (d) \(-\frac{29}{25} \)

8. (a) \( (x^3 \cos x)' = (x^3)' \cos x + x^3(\cos x)' = 3x^2 \cos x - x^3 \sin x \)

   (b) \( \frac{1+\cos x + \sin x}{(1+\cos x)^2} \)

   (c) \( e^x (\tan x + \sec^2 x) \)
(d) \[ \frac{\sec x \tan x - (\sec x)/2}{x\sqrt{x}} \]

9. \( y = \frac{1}{2} \)

10. \(-2\sin x.\)

11. (a) 4
  (b) \( \frac{7}{5} \)