Math 1131 Week 4 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

3.1: Derivatives of Polynomials and Exponential Functions

1. Use differentiation rules from Section 3.1 (**not other methods**) to compute the derivative of the following functions.

   (a) $f(x) = 7x^3 - 5x + 8$

   (b) $f(x) = e^x + x^e$

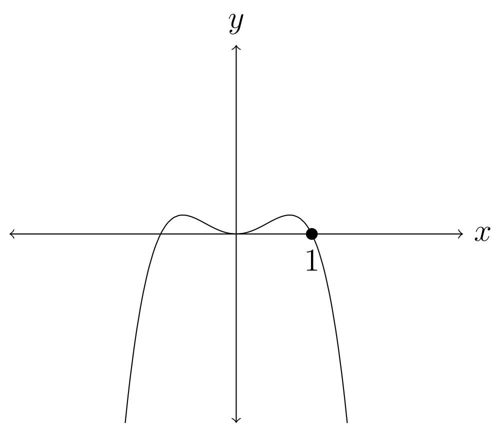
   (c) $f(x) = 3x + \sqrt{3x}$

   (d) $f(x) = \sqrt[4]{x} - 4e^x$

   (e) $f(x) = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

   (f) $f(x) = \dfrac{12}{x^5} - \dfrac{7}{\sqrt[5]{x}}$

2. Use differentiation rules to find the equation of the tangent line to $y = x^2 - x^4$ (see below) at the point $(1, 0)$.



3. Use differentiation rules to find the values of a and b that make the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2, \\ ax^3 + bx & \text{if } x > 2 \end{cases}$$

differentiable at $x = 2$.

4. Find all points $(c, f(c))$ on the graph of $f(x) = x^3 - 3x^2$ where the tangent line has slope 9.

5. T/F (with justification) If $f(x) = \sqrt{7}$ for all x, then $f'(x) = \dfrac{1}{2\sqrt{7}}$ for all x.

3.2: The Product and Quotient Rules

6. Compute the derivative of each function below using the methods from Sections 3.1 and 3.2 (**not other methods**).

   (a) $f(x) = \dfrac{x}{x+3}$ (simplify numerator in final answer)

   (b) $f(x) = \dfrac{e^x}{1+e^x}$ (simplify numerator in final answer)

   (c) $f(x) = \sqrt{x}e^x$

(d) $f(x) = \dfrac{e^x}{x^n}$ for constant n, in two ways: (i) quotient rule and (ii) product rule

(e) $f(x) = \dfrac{1}{x} + \dfrac{1}{1-x}$ (in final answer, use a common denominator and simplify numerator)

7. In the function $h(x)$ below, defined in terms of $f(x)$ and $g(x)$, determine $h'(2)$ in each case if $f(2) = 3$, $g(2) = 4$, $f'(2) = 1$, and $g'(2) = -5$.

   (a) $h(x) = 2f(x) + 5g(x)$

   (b) $h(x) = f(x)g(x)$

   (c) $h(x) = \dfrac{f(x)}{g(x)}$

   (d) $h(x) = \dfrac{g(x)}{f(x) + 2}$

3.3: Derivatives of Trigonometric Functions

8. Compute the derivative of each function below using differentiation rules.

   (a) $f(x) = x^3 \cos x$

   (b) $f(x) = \dfrac{1 + \sin x}{1 + \cos x}$

   (c) $f(x) = e^x \tan x$

   (d) $f(x) = \dfrac{\sec x}{\sqrt{x}}$ (Compute (d) in **two ways**, using (i) the quotient rule and (ii) the product rule.)

9. Find the equation of the tangent line to the curve $y = \sin x \cos x$ at $x = \dfrac{\pi}{4}$. (Your coefficients must be exact, not approximations.)

10. Find the higher derivative $\dfrac{d^{1881}}{dx^{1881}}(2\cos x)$ by finding the first eight derivatives and observing the pattern that occurs.

11. Determine the following limits by making a change of variables to allow you to use the relation $\lim_{t \to 0} \frac{\sin t}{t} = 1$.

(a) $\lim_{x \to 0} \frac{\sin 4x}{x}$

(b) $\lim_{x \to 0} \frac{\sin 7x}{5x}$

Answers to selected problems

1. (a) $f'(x) = 21x^2 - 5$
   (b) $f'(x) = e^x + ex^{e-1}$
   (c) $f'(x) = 3 + \dfrac{\sqrt{3}}{2\sqrt{x}}$
   (d) $f'(x) = \dfrac{1}{4x^{3/4}} - 4e^x$.
   (e) $f'(x) = \dfrac{3}{2}\sqrt{x} + \dfrac{2}{\sqrt{x}} - \dfrac{3}{2x^{3/2}}$.
   (f) $f'(x) = -\dfrac{60}{x^6} + \dfrac{7}{5x^{6/5}}$.

2. $y = -2x + 2$

3. $a = 1/4$, $b = 1$.

4. $(-1, f(-1)) = (-1, -4)$ and $(3, f(3)) = (3, 0)$.

5. False

6. (a) $\frac{3}{(x+3)^2}$
   (b) $\frac{e^x}{(1+e^x)^2}$
   (c) $\frac{(2x+1)e^x}{2\sqrt{x}}$
   (d) (i): $\frac{x^n e^x - e^x n x^{n-1}}{x^{2n}}$
   (ii): $e^x x^{-n} - n e^x x^{-n-1}$ (show why these are the same!)
   (e) $\frac{2x-1}{x^2(1-x)^2}$

7. (a) -23
   (b) -11
   (c) $\frac{19}{16}$
   (d) $-\frac{29}{25}$

8. (a) $(x^3 \cos x)' = (x^3)' \cos x + x^3 (\cos x)' = 3x^2 \cos x - x^3 \sin x$
   (b) $\frac{1+\cos x+\sin x}{(1+\cos x)^2}$
   (c) $e^x(\tan x + \sec^2 x)$

(d) $\frac{x\sec x\tan x-(\sec x)/2}{x\sqrt{x}}$

9. $y=\frac{1}{2}$

10. $-2\sin x$.

11. (a) 4

    (b) $\frac{7}{5}$