

Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

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**Solutions should show all of your work, not just a single final answer.**

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## 3.4: The Chain Rule

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1. Compute the derivative with respect to  $x$  of each function below using differentiation rules.

(a)  $f(x) = (x^3 - x + 1)^{10}$

(b)  $f(x) = \sqrt{x^3 + 4x}$

(c)  $f(x) = e^{ax} \cos(bx)$  for constants  $a$  and  $b$

(d)  $f(x) = \left(\frac{e^x}{3-x}\right)^8$

(e)  $f(x) = \sin^2(x) - \sin(x^2)$

2. Differentiate the functions below **with respect to**  $t$ , where  $r = r(t)$  is a function of  $t$ .

(a)  $(r^2 + 1)^4$

(b)  $\sin(2r) - 2 \sin r$

(c)  $e^{r^2+ar+b}$  for constants  $a$  and  $b$ .

3. If  $f'(0) = 5$  and  $F(x) = f(3x)$ , what is  $F'(0)$ ?

4. T/F (with justification) If  $f(x)$  is differentiable, then  $\frac{d}{dx}(f(\sqrt{x})) = \frac{f'(x)}{2\sqrt{x}}$ .

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## 3.5: Implicit Differentiation

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5. Find  $\frac{dy}{dx}$  using implicit differentiation. Your final answer may involve both  $x$  and  $y$ .

(a)  $x^2y - axy^2 = x + y$  where  $a$  is a constant.

(b)  $\sin(x + y) = x + \cos(3y)$

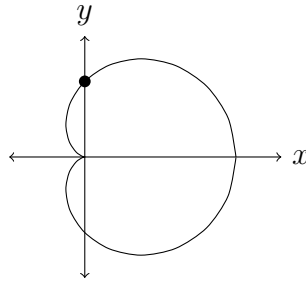
(c)  $e^{xy} = x^2 + y^2$

(d)  $x = \arctan(y^2)$

6. Use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

at the point  $(0, 1/2)$ . **Note.** The graph of this equation is known as a cardioid, shown below. It's not the graph of a function, and this is where implicit differentiation can be helpful to us.



7. On the ellipse  $x^2 + 9y^2 = 9$ , find  $\frac{d^2y}{dx^2}$  using implicit differentiation. Your final answer may involve both  $x$  and  $y$ .

