Math 1131 Week 10 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

4.7: Optimization Problems

1. A closed box (top, bottom, and all four sides) needs to be constructed to have a volume of $9\,\mathrm{m}^3$ and a base whose width is twice its length. See Figure 1.

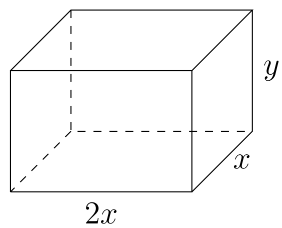

Figure 1: A box

Use calculus to determine the dimensions (length, width, height) of such a box that uses the least amount of material. Justify why your answer corresponds to a minimum, not a maximum.

2. We want to find the points on $y = x^2$ that are closest to $(0, 3)$.

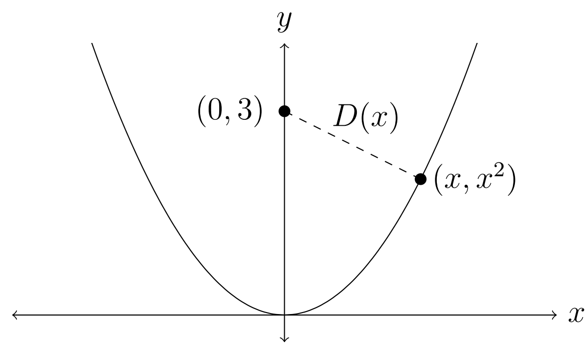

Figure 2: Distance to $(0, 3)$ on $y = x^2$.

(a) For each point (x, x^2) on the parabola, find a formula for its distance to $(0, 3)$. Call this distance $D(x)$. (See Figure 5.)

(b) Let $f(x) = D(x)^2$, which is the *squared distance* between (x, x^2) and $(0, 3)$. Finding where $D(x)$ is minimal is the same as finding where $f(x)$ is minimal. Determine all x where $f(x)$ has an absolute minimum. The points (x, x^2) for such x are the closest points to $(0, 3)$ on $y = x^2$.

3. Three line segments of length 1 are joined together at endpoints to form a base and the legs of an isosceles trapezoid, as in Figure 3. Let θ in $(0, \pi/2)$ be the common angle measurement between the legs and the line passing through the base of length 1. We want to find the angle θ that maximizes the area of the trapezoid.

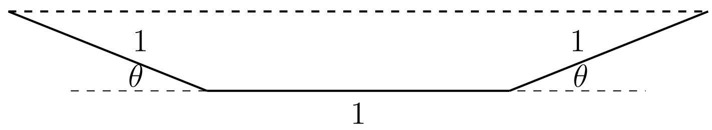

Figure 3: An isosceles trapezoid with base and legs of length 1.

(a) Compute the area $A(\theta)$ of the trapezoid. The general area formula for a trapezoid is $\frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of the bases. (Hint: Break up the trapezoid into a rectangle with two right triangles at both ends. Use trigonometry to compute the height and the length of the longer base in terms of θ.)

(b) Find all solutions to $A'(\theta) = 0$ with $0 < \theta < \pi/2$. (The answer is *not* $\pi/4 = 45^o$.)

(c) Verify that the area $A(\theta)$ is a maximum, not a minimum, at the angle found in part (b) and compute this maximum area.

4.8: Newton's Method

4. Apply Newton's method to estimate the solution of $x^3 - x - 1 = 0$ by taking $x_1 = 1$ and finding the least n such that x_n and x_{n+1} agree to three digits after the decimal point.

5. The number π is a solution of $\sin x = 0$ close to 3 (see Figure 4). You will use Newton's method for $\sin x = 0$ to create numerical estimates for π.

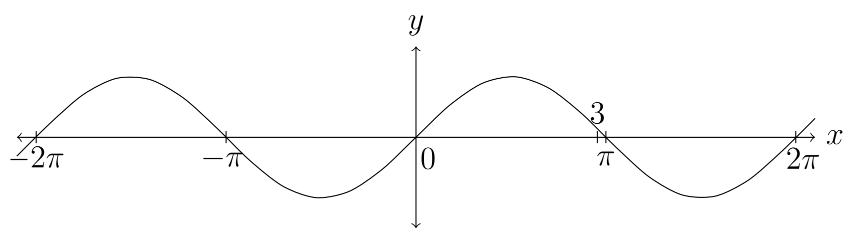

Figure 4: Graph of $y = \sin x$.

   (a) Write out the recursion for Newton's method used to solve $\sin x = 0$.

(b) Using Newton's method for $\sin x = 0$ with $x_1 = 3$, find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point. (Use radians, *not* degrees!)

(c) For the n you found in part (b), to how many digits after the decimal point does x_n actually agree with π?

6. In Figure 5 is the graph of $f(x) = \ln(x) - 1$ for $0 < x < 4$. It crosses the x-axis at $x = e$. You will use Newton's method for $f(x) = 0$ to create numerical estimates for e.

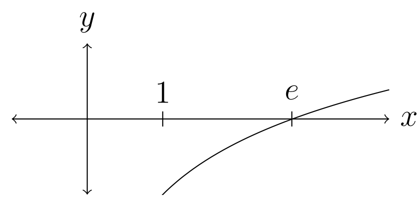

Figure 5: Graph of $y = \ln(x) - 1$.

(a) Using Newton's method for the equation $\ln(x) - 1 = 0$ with $x_1 = 1$, tabulate x_n to find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point.

(b) For the n you found in part (a), to how many digits after the decimal point does x_n actually agree with e?