Points and Surfaces in Different Coordinate Systems

This week, we will be looking at the rectangular, cylindrical, and spherical coordinate systems.

Rectangular Coordinates

Rectangular coordinates, also known as Cartesian coordinates, is a system you are very familiar with at this point. Points in this coordinate system are described by the three variables x, y, and z. Each variable defines the distance of the point from its respective coordinate plane. For example, the point (1,2,3) is 1 unit from the x=0 plane, 2 units from the y=0 plane, and 3 units from the z=0 plane.

1. [Click here to access the GeoGebra demo for this section](https://www.geogebra.org/3d/jttu4qcw) (https://www.geogebra.org/3d/jttu4qcw)
2. This is a basic demo to show the principle idea behind rectangular coordinates.
3. Move the sliders to change the values of x, y, and z. Notice that the shape of the object produced by the point is always a rectangular prism. In two dimensions, what shape would this coordinate system make? (Set one of the variables to zero to see!)
4. Think about what surfaces are easy to describe in rectangular coordinates. What are the strengths of cartesian coordinates. What are the weaknesses? Keep these questions in mind when exploring the other coordinate systems. Remember that coordinate systems are tools that we have invented to help us describe objects in 3D space.

Relating the Coordinate Systems

1. [Click here to access the GeoGebra demo for this section](https://www.geogebra.org/3d/sx8jmmqs) (https://www.geogebra.org/3d/sx8jmmqs)
2. In this demo, you will be converting cylindrical and spherical coordinates to Cartesian form, and determining the signs of x, y, and z.
3. Refer to the textbook or your notes on how to convert between rectangular, cylindrical, and spherical coordinates. Recall that theta is the angle in the xy-plane, counter-clockwise from the positive x axis. Phi is the angle of declination from the positive z axis.
4. Use your knowledge to convert the points A through F into rectangular form. You can find these points on the sidebar in the GeoGebra demo.
5. When you would like to check your work, use the Xpositive, Ypositive, and Zpositive checkboxes to indicate the sign of each variable for point A. When a variable is positive, you should make it so that the checkbox reads true. If the variable is negative, make the checkbox false. Then, click the showA checkbox. If you have the correct signs, the point on your screen will be green. If you are incorrect, the point will be red. Uncheck showA and proceed to the next step.

6. Repeat this process for the rest of the points B through F, until you are able to get a green point for each attempt.

Spherical Coordinates

1. [Click here to access the GeoGebra demo for this section](https://www.geogebra.org/3d/msvvdbvn) (https://www.geogebra.org/3d/msvvdbvn)
2. This is a basic demo to show the principle idea behind spherical coordinates. In the demo you'll find a RhoValue slider, and buttons to trigger the appearance of Rho, Theta, and Phi.
3. Move around the RhoValue slider to see the effect of changing that value. What happens when you increase it? Decrease it? The sphere's size should increase and decrease respectively. Rho is the distance from the origin to a point, and generally, is essentially the radius of the sphere centered at the origin that this point would sit on.
4. Turn on the ShowTheta button, then begin to move PointA around. You should see a "Theta =" value. To understand what this value represents, imagine this: project this point down to the xy-plane. Basically, thinking in Cartesian coordinates, make the z-coordinate 0. The angle that is made from the positive x-axis, the origin, and that point in order represents the Theta value. Turn on PointProjectedOnXYPlane to visualize this idea. Usually, it will be written in radian form, but this demo will show it in degrees. In addition, the Theta value in this demo maximizes at 180 degrees (π), and going past this value just goes back to 0, which is not representative of how Theta truly acts (it goes from 0 to 360 degrees/2π).
5. Then, turn on the ShowPhi button, and keep moving around PointA. You should see a "Phi =" value that changes as the point moves. This value represents the angle between the z-axis and the line connecting the origin and the point. Once again, this value is usually in radians, but the demo shows it in degrees, and the demo maximizes at 180 degrees (π). In this case, this maximum works out because when you think about Phi, it actually creates a cone in spherical coordinates. The angle goes in all directions revolving around the z-axis.
6. Continue to move around PointA to see the values change, and feel free to toggle certain buttons on and off.

Cylindrical Coordinates

Cylindrical coordinates is a system that you may not be all too familiar with because it is quite new. The goal of this demo is to get you to have a better understanding of the coordinate system as a whole, with a focus on the z, r, and θ values. This demo also does reference question number 2 on Worksheet 6 with hopes of providing you with a nice visual.

1. Click on this link to access the demo (https://www.geogebra.org/3d/u8zjcnwt)
2. When you open the demo, you will see a point A on a cylinder. There are sliders that you can adjust if you want to see how changing the z, r, and θ values will affect the location of this point.
   a. What happens to the point when you change the z values?
   b. What happens to the point when you change the r values?
   c. What happens to the point when you change the θ values?
3. Let's investigate what happens when r is kept constant. Choose an r value with the slider and then adjust the other two sliders however you would like. Do you notice that no matter how you change θ and z, the point will always be on the cylinder? Thus, when the r value is kept constant, a cylinder will result.
4. Now let's look at what would happen when z is kept constant. Deselect the checkbox that says "ShowConstantR" and select the checkbox that says "ShowConstantZ." This will show a horizontal plane at a z value that you can choose with the slider. This value will be kept constant as you adjust the values for r and θ. What do you notice? Do you see that no matter how r and θ are adjusted, the point will always be on the plane? Thus, when the z value is kept constant, a plane parallel to the xy plane at the given z value will result.
5. Lastly, let's look at what would happen when θ is kept constant. Deselect the checkbox that says "ShowConstantZ" and select the checkbox that says "ShowConstantθ." This should show a "half-plane." Set the θ value to .79(which is about $\frac{\pi}{4}$). Now, change the z and r values with the sliders as you please. Notice that no matter what changes are made to r and z, the point will always lie on the half-plane in the $\frac{\pi}{4}$ direction(which is the angle from the positive x-axis). This is only a half-plane that starts at the z-axis because if it were to go past the z-axis, then the plane would be representative of 2 different values of θ: $\frac{\pi}{4}$ and $\frac{\pi}{4} + \pi$ (which is also $\frac{5\pi}{4}$). Thus, a constant theta would lead to a half plane in the direction of the θ value.

A Visual for a 15.7 WebAssign Question

On the 15.7 WebAssign, you are asked to find the area of the region that is contained in both a sphere and a cylinder. This small model just gives a brief visual of the region that you were asked to find the volume of. This model gives a visual for the volume of a region contained in the cylinder $x^2 + y^2 = 25$ and the sphere $x^2 + y^2 + z^2 = 81$.

1. Use this link to access the demo (https://www.geogebra.org/3d/cqtrkny8)
2. You can click the "showsphere" checkbox to see the sphere and click the "showclinder" checkbox to see the cylinder.
3. That are the two solids given in the problem. To see the actual region that you are asked to find the volume of, click the checkbox "showregion" and deselect the other two checkboxes.