Cylinders, Quadric Surfaces and Their Traces

In this activity, we will be exploring some of the different cylinders and quadric surfaces that exist in 3D, as well as analyzing their traces with different planes. Each family of surfaces will have a GeoGebra Demo provided.

Cylinders

A cylinder is a surface that is made up of all the possible lines (also called rulings) that are 1) parallel to a given line and 2) pass through a given plane curve. If a variable (x, y, or z) is missing from a surface's equation, it's an indication that the surface is a cylinder.

1. Click here to visit the demo for cylinders (https://www.geogebra.org/3d/mwtdrpct)
2. To start, turn on the function for *ParabolicCylinder, p : z = k,* and *c : IntersectPath* by clicking the buttons to the far left, if they aren't on already. You should see a parabolic cylinder and its trace in the z-plane.
3. Try moving the slider for *k*. Does the trace change at all? Why or why not?
4. The z-trace for $y = x^2$ in 3D should always be the same because z is missing from the equation, and so it is a constant. The parabola exists for every z-value. Turn off the function for *ParabolicCylinder* and *c : IntersectPath*, leaving *p : z = k* on.
5. Next, turn on the function for *Cylinder* as well as *d : IntersectPath*. Here you'll see another example of a cylinder, whose trace is a circle because the *a* and *b* values are the same. Since once again the z-value is the missing variable, z is a constant, and so the traces in the z-plane will all be the same. To move the trace around, scroll back up and move around the slider for *p : z = k*.
6. Now try moving the sliders for *a* and *b* to different values. How does this change the surface? You'll notice that when $a > 0$, $b > 0$, and $a \neq b$, the result is an elliptic cylinder. When *a* and *b* are opposite signs, the result is a hyperbolic cylinder. When $a = 0$, $b = 0$, or $a < 0$ and $b < 0$, there is no graph. Take a look at the Cylinder equation to see why this might be (hint: when *a* and *b* are at these values, can the equation equal 1?). After you are done, turn off all buttons.
7. Turn on the function for *SineWaveCylinder, q : x = j,* and *eq1 : IntersectPath*. This time x is the missing variable. What does this mean in terms of how the graph will look? To view the trace of the function, try moving around the *j* slider.
8. Since x is missing, it is a constant, and so all of the traces along the x-plane will be the same.

Ellipsoids

An ellipsoid is a quadric surface that looks like a stretched out sphere. Just as an ellipse corresponds to a circle in 2D, an ellipsoid corresponds to a sphere in 3D.

1. Click here to visit the demo for ellipsoids (https://www.geogebra.org/3d/shbsygka)
2. Click inside the box that says *ellipsoid*. Here you can see the general form equation of an ellipsoid.
3. Try moving the sliders for *a*, *b*, *c*, and *d* to different values. How does this change the shape of the surface? (Hint: the first three letters "stretch" the ellipsoid in different directions)
4. When you are done with the first set of sliders, click the small circle next to the boxes titled *plane* and *intersection*. This will allow you to move and rotate a plane in 3D space and view the intersection of this plane and the ellipsoid.
5. Try moving the sliders for *A*, *B*, *C*, and *D* to edit the plane. Look at the intersections you create (if you'd like to take a better look, you may turn the plane on and off). You may notice that every intersection is an ellipse.
6. Move the sliders so that $a=b=c$. You've just made a special form of an ellipsoid—the sphere. What shape do the intersections make now?

Cones

1. Click here to visit the demo for cones (https://www.geogebra.org/3d/d4u3avdv)
2. When you click to open the document, you will see a true cone. Take a look at the general form of this type of surface by clicking inside the box title "TrueCone".
3. Try playing with the sliders for *a, b,* and *c*. What effect does switching these values have on the shape of the surface?
4. Now that we have done some experimenting with the values of *a, b,* and *c*, we can take a look at some of the traces. In order to view the traces, turn on the intersection labeled "ytraces1" to see the y-traces, "xtraces1" to view the x-traces, etc.(this will take a little scrolling down). You can view these traces one at a time or all at once if you please.
5. Above that, there are sliders that you can move to see how the traces will change. Make sure to take note of the different shapes that each of these traces have.
    a. The slider for *d* moves the z-traces
    b. The slider for *f* moves the y-traces
    c. The slider for *g* moves the x-traces
6. You can also adjust the sliders for *a, b,* and *c* to change the shape of the true cone to see how that will affect the shape of the traces.
7. Next, we will investigate a similar type of cone: a half cone. First, hide the true cone that you were previously looking at(labeled "TrueCone") and hide the traces as well. Now scroll down and look for the new cone, named "SingleCone(x,y)" and toggle it on so that it is showing on the graph.

8. Take a look at the shape of this different type of cone and the equation for it as well. What differences can you find?
9. We can now change sliders *a* and *b* and see how this can affect the shape of this half cone.
10. Then, we can look at the traces. This time, the traces are labeled as “ytraces2” for y-traces, “xtraces2” for x-traces, and “ztraces2” for z-traces. Like before, you can look at one trace at a time or all at the same time.
11. You can toggle the sliders mentioned in Step 5 to move the traces around. Take note of the shape of these traces. What is different about these traces when compared to those of the true cone?
12. Lastly, sliders *a* and *b* can be changed to see how that would affect the shape of the traces.

Elliptic Paraboloids

Elliptic Paraboloids are a family of quadric surfaces whose traces parallel to one coordinate plane (the variable raised to the first power) are ellipses (in special cases they’re circles), and whose traces parallel to the other two coordinate planes are parabolas.

1. [Click here to visit the demo for elliptic paraboloids](https://www.geogebra.org/3d/fsdursjk) (https://www.geogebra.org/3d/fsdursjk)
2. The general form of an elliptic paraboloid is shown below:
$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
3. Turn on eq1 if it isn’t already. Try moving around the sliders for *a*, *b*, and *c* to change the shape of the surface. What does each value do to the graph of the function?
4. To see the trace of the z-plane, turn on p : z = k and d : IntersectPath. Try moving the slider for k to see the trace at different z-values. What shape is this trace? Does it change with varying *a*, *b*, and *c* values?
5. Turn off p : z = k and d : IntersectPath. To see the trace of the x-plane, turn on r : x = q and e : IntersectPath. Try moving the slider for q to see the trace at different x-values. What shape is this trace? Does it change with varying *a*, *b*, and *c* values?
6. Turn off r : x = q and e : IntersectPath. To see the trace of the y-plane, turn on s : y = w and f : IntersectPath. Try moving the slider for w to see the trace at different y-values. What shape is this trace? Does it change with varying *a*, *b*, and *c* values?
7. If you haven’t already, set *a*=*b* and turn on p : z = k and d : IntersectPath. Feel free to try playing with the k-slider. When *a*=*b*, the z-trace is a circle.

Hyperbolic Paraboloids

Hyperbolic Paraboloids are a unique family of quadric surfaces. They are known for their "saddle point" and look like the shape of Pringles chips.

1. Click here to visit the demo for hyperbolic paraboloids (https://www.geogebra.org/3d/nybjr2bp)
2. The general form of a hyperbolic paraboloid is listed below as equation a:
    a. $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
3. This equation is rearranged in GeoGebra so that it is a function of two variables.
4. Try moving the sliders for *a*, *b*, and *c* to change the shape of the surface. What does each value do to the function?
5. When you are ready to see the traces of the function, you may use the ShowTrace checkboxes to toggle on or off each trace. Use the sliders for *X*, *Y*, and *Z* to move the traces around.
6. The horizontal traces and vertical traces form different 2D graphs. What are the names of the graphs that these traces form? You may wish to zoom in and out with the scroll wheel to get a better view of the function and traces. (Hint: The traces are where this function gets its name from. Try removing a variable from the equation to find its trace.)

Hyperboloids

1. Click here to visit the demo for hyperboloids (https://www.geogebra.org/3d/uajudraq)
2. When you click to open the document, you will see a hyperboloid of one sheet. Take a look at the general form of this type of surface, which is listed right after "OneSheetHyperboloid"
3. Try playing with the sliders for *a, b,* and *c*. What effect does switching these values have on the shape of the surface?
4. Now that we have done some experimenting with the values of *a, b,* and *c*, we can take a look at some of the traces. In order to view the traces, turn on the intersection labeled "ytraces1" to see the y-traces, "xtraces1" to view the x-traces, etc.(this will take a little scrolling down). You can view these traces one at a time or all at once if you please.
5. Above that, there are sliders that you can move to see how the traces will change. Make sure to take note of the different shapes that each of these traces have.
    a. The slider for *d* moves the z-traces
    b. The slider for *f* moves the y-traces
    c. The slider for *g* moves the x-traces
6. You can also adjust the sliders for *a, b,* and *c* to change the shape of the hyperboloid to see how that will affect the shape of the traces.
7. Next, we will investigate a different type of hyperboloid: one of two sheets. First, hide the hyperboloid of one sheet that you were previously looking at(labeled

“OneSheetHyperboloid”) and hide the traces as well. Now scroll down and look for the new hyperboloid, named “TwoSheetHyperboloid” and toggle it on so that it is showing on the graph.

8. Take a look at the shape of this different type of hyperboloid and the equation for it as well. What differences can you find?
9. Once again, we can play with the above sliders of *a, b,* and *c*, like we did with the previous hyperboloid. Take note of how changing these sliders affect the shape.
10. Then, we can look at the traces. This time, the traces are labeled as “ytraces2” for y-traces, “xtraces2” for x-traces, and “ztraces2” for z-traces. Like before, you can look at one trace at a time or all at the same time.
11. You can toggle the sliders mentioned in Step 5 to move the traces around. Take note of the shape of these traces. Notice how there are no z-traces in between the two sheets. Take a look at the equation to see why this might be (Hint: plug values into the equation to see why z must be outside a certain range).
12. Lastly, feel free to adjust the *a, b,* and *c* sliders to see how that will affect the shape of the traces.