Partial Derivatives and Tangent Planes

Over the past few classes, we have been working with partial derivatives and tangent planes. In this demo, we will be further exploring these topics by visualizing them, rather than just working with the numbers. This demo consists of some of the surfaces that were included in last week's demo, as well as some other functions as well.

Paraboloids

1. Now that you have a stronger understanding of the different quadric surfaces and their traces, let's dive a little deeper! This first demo involves paraboloids. [Click here to access this demo.](https://www.geogebra.org/3d/rqfs5bny) (https://www.geogebra.org/3d/rqfs5bny)
2. Once the demo is open, turn on the Paraboloid function if it is not on already. If you'd like to, play around with the *a* and *b* sliders as you please until you find a shape that you want to work with.
3. Notice the Point A sitting on the surface of this function. You may move this point around as you please. You will soon see the relevance and importance of this point.
4. Turn on the TangentPlane function. Begin to move the point around again. How does the TangentPlane change in response to point A's movements?
5. To view the x and y traces of the function, turn on XTrace and YTrace. You can keep these on to help visualize what's going on if you'd like.
6. Click the circle next to NormalVector to view the normal vector to the function at point A. Move around the point to see how this vector changes.
7. To view the Tangent lines to the function at point A, turn on the two Tangent lines near the bottom. These lines represent the slopes of the partial derivatives of the function at point A with respect to x and y respectively. Notice that the line goes both directions. The direction of the derivative is influenced by the direction vector needed to calculate directional derivatives.
8. It's time to experiment! Toggle on and off whichever elements you'd like, and continue to move the point around on the function's surface.

Hyperboloids

1. Last week, you took a brief look at hyperboloids and their traces. This week, we will be taking a deeper dive into hyperboloids by looking at partial derivatives and tangent planes. [Click here to access this demo.](https://www.geogebra.org/3d/jwdgaxqa) (https://www.geogebra.org/3d/jwdgaxqa)
2. When you first open the demo, you will see a hyperboloid of one sheet written as:
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$
3. Feel free to adjust the sliders *a* and *b* to change the way the hyperboloid looks. You can also move the point "A" that is on the surface around as well.
4. To see a plane tangent to point A on the hyperboloid, toggle on the equation labeled "TangentPlane"
5. You can toggle on the "xtrace" and "ytrace" as well if you would like to see those at the point.
6. Below that, there is an option to toggle on a "NormalVector." This vector is perpendicular to the tangent plane at the point "A".
7. Another thing that you can do is look at the partial derivatives. The next two lines of equations will show lines of the x and y partial derivatives. These lines will show the rate of change of the function in the x and y directions at the point on the hyperbola.
8. At this point, you can toggle whatever you want on and off. Try and move the point around to see how the tangent plane and partial derivatives will change at different points on the surface. Feel free to toggle some of the previous options off if you find the demo to be too crowded and/or confusing at this time.
9. You can also experiment with a hyperboloid of two sheets. Just replace the equation in step 2 with the equation:
$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
10. Once you change the equation, you can go through steps 3-8 again if you so choose.

General Surfaces

1. Now that you have some experience with tangent planes and partial derivatives, feel free to experiment with the final demo. Click here to access this demo. (https://www.geogebra.org/3d/jvjgaxkc)
2. With this demo, you may type any function of two variables into the box labeled *f(x,y)*. The default function is a semi-ellipsoid, but you may edit this however you'd like. Then, you can use the *X* and *Y* sliders or type in values manually to move the point P. You may also drag the point. Below the sliders are the numerical values of the partial derivatives.
3. By default, the traces and partial derivative vectors are turned off. Use the checkboxes to toggle these visualizations on and off. You may also turn the tangent plane on and off by clicking the circle next to the label *TangentPlane*. Lastly, you can adjust the scale of the partial derivative vectors by using the *ScaleVectors* slider.
    a. The black vector is the normal vector of the tangent plane. The red and green vectors show the direction of the tangent line for the x and y partial derivatives, respectively.
4. Use this demo to view the tangent planes, traces, and partial derivative vectors of various functions. Listed below are some two-variable functions that you may experiment with.
    a. $z = \sqrt{x^2 + y^2}$ (a cone)
    b. $z = sin(x)$ (a cylinder)
    c. $z = xy^2$
    d. $z = x/y$
5. For any of these functions, you may toggle the checkboxes to improve visibility. A good exercise to improve understanding is to try to predict the shape of the trace or the sign of a partial derivative at a certain point. Then, use the demo to check your prediction.