Section 6.1: Area Between Curves

(1) In this section, we use integration to find the area of regions bounded by curves. Explain, in terms of adding up the area of rectangles, how this works.

> Let's define the region as S. We divide S into n strips of equal width and then we approximate the i th strip by a rectangle with base Δx and height $f(x_i^*) - g(x_i^*)$, the Riemann sum
>
> $$\sum_{i=1}^{n}[f(x_i^*) - g(x_i^*)]\Delta x$$
>
> is therefore an approximation to what we intuitively think of as the area of S.
>
> This approximation will become better if n $\to \infty$. Therefore we can define the **area** A of the region S as the limiting value of the sum of the areas of these approximating rectangles.
>
> $$A = \lim_{x\to 0}\sum_{i=1}^{n}[f(x_i^*) - g(x_i^*)]\Delta x$$

(2) Explain how it works in terms of thinking about the area under the first curve minus the area under the second.

> By looking at the area integral formula, i.e, $A = \int_a^b [f(x) - g(x)]dx$, and assume $f(x) \geq g(x)$ for all x in [a,b]. The area under the first curve is $\int_a^b f(x)dx$ and the area under the second curve is $\int_a^b g(x)dx$, which is directly coming from the definition of integral. Then the formula tells us the interpretation that the region's area equals the area under the first curve minus the area under the second curve.

(3) When finding the area between two curves, does the area below the x-axis count as negative? Explain your reasoning.

> No, based on the area integral formula as $A = \int_a^b [f(x) - g(x)]dx$, if we assume $f(x)$ is 0 and $g(x)$ is negative on the interval [a,b], then our area integral formula will just be $A = \int_a^b -g(x)dx$.And the integral of $\int_a^b g(x)dx$ will be negative. However, from the area integral formula, we have this part as positive, since we put a negative sign in the front to make the area as a positive number.

(4) When we integrate with respect to x, which curve is first (which to we subtract from)? What about when we integrate with respect to y?

When we integrate with respect to x, let's assume there are two functions, $f(x)$ and $g(x)$, we will integrate $f(x)$ first if $f(x) \geq g(x)$. However, sometimes the whole domain of our integral need to be split into several intervals, as $f(x)$ doesn't dominate on the whole domain. Then the larger function on each interval will be integrated first.

When we integrate with respect to y, we will get a function by regarding x as a function of y firstly, then if $f(y) \geq g(y)$, we will integrate $f(y)$ first and we still need to split the region into several intervals if necessary. Thus we integrate right (larger value) minus left=(smaller value).

(5) Sketch examples of regions bounded by curves that satisfy each of the following:

(a) Area can only be found with one integral by integrating with respect to x.

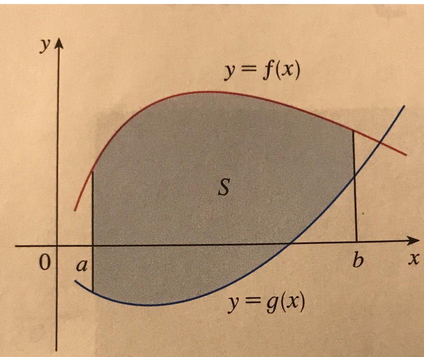
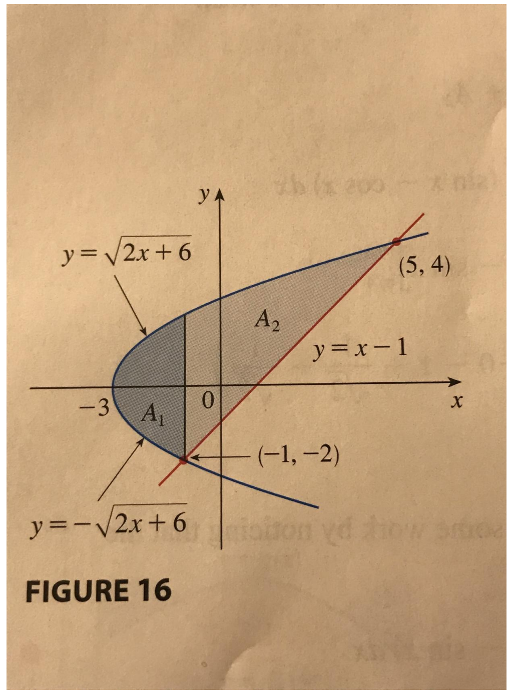
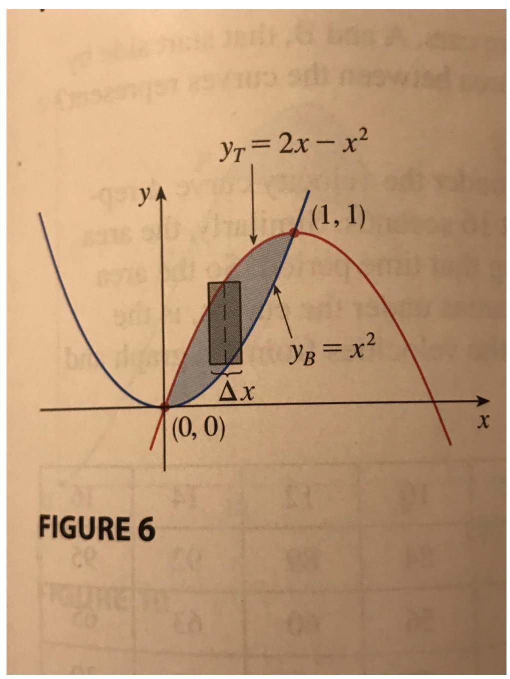


FIGURE 1
$S = \{(x, y) \mid a \leqslant x \leqslant b, g(x) \leqslant y \leqslant f(x)\}$

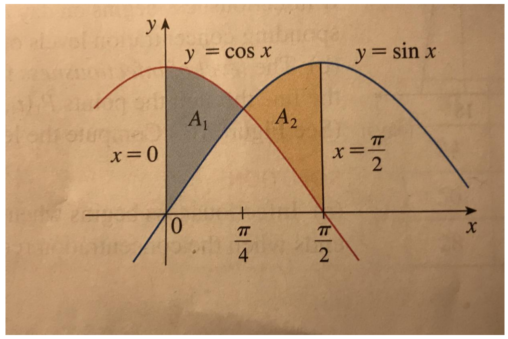
(b) Area can only be found with one integral by integrating with respect to y.



(c) Area can be found with one integral by integrating with respect to x or y.



(d) We must use two integrals if we integrate with respect to x and at least two if we integrate with respect to y.



Extra Practice in Book: 6.1: 3, 9, 11, 13, 17, 23, 33