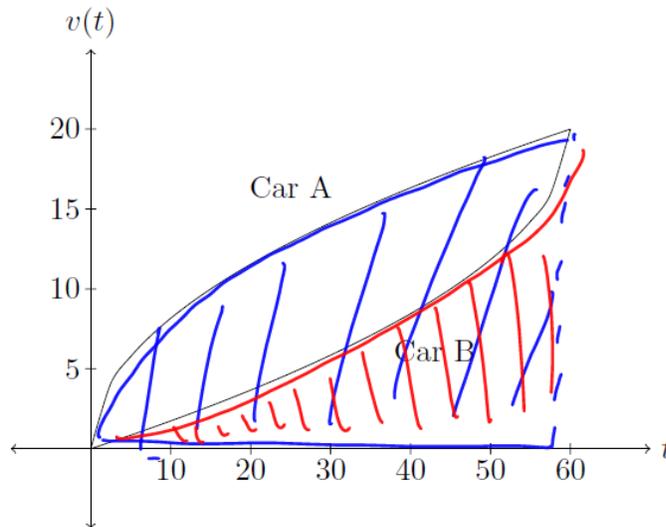


# MATH 1131Q Final Exam Practice Questions - Solutions

1 – Be sure to review Exams 1 and 2 and their practice sets, as well as other materials like worksheets and quizzes!

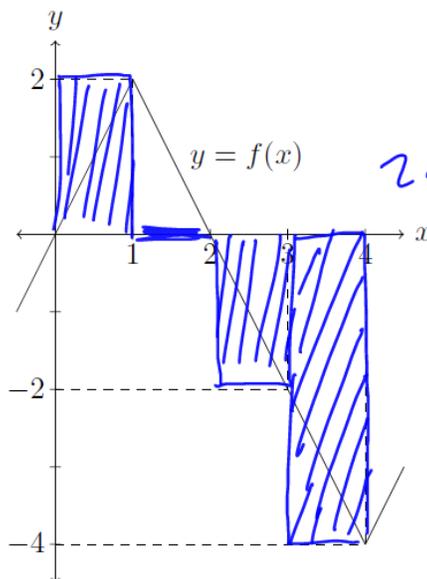
- 2 Below is the graph of the velocity (measured in ft/sec) over the interval  $0 \leq t \leq 60$  for two cars, Car A and Car B. How do the distances traveled by each compare at over this interval?



Car A has traveled further than Car B (blue area bigger than red)

- 3 If we use a right endpoint approximation with four subintervals (i.e.,  $R_4$ ), then what is the resulting approximation for

$$\int_0^4 f(x) dx?$$



$$2 + 0 - 2 - 4 = -4$$

4

Evaluate the definite integral  $\int_{-1}^1 (x^2 + 2x + 1) dx$ .

$$= \left[ \frac{x^3}{3} + x^2 + x \right]_{-1}^1$$

$$= \left( \frac{1}{3} + 1 + 1 \right) - \left( -\frac{1}{3} + 1 - 1 \right)$$

$$= \frac{2}{3} + 2 = \underline{\underline{\frac{8}{3}}}$$

5

Assume that  $\int_{-2}^3 f(x) dx = 4$ . What is the value of  $\int_{-2}^3 (f(x) + 1) dx$ ?

(A) 4      (B) 5      (C) 6

(D) 9      (E) 20

$$= \int_{-2}^3 f(x) dx + \int_{-2}^3 1 dx$$

$$= 4 + 1(3 - (-2))$$

$$= 4 + 5 = \underline{9}$$

6

Which of the following is the derivative of the function

$$f(x) = \int_1^{x^2} \frac{1}{t^3 + 1} dt?$$

(A)  $\frac{2x}{x^6 + 1}$

(B)  $\frac{1}{x^6 + 1}$

(C)  $\frac{2x}{x^5 + 1}$

(D)  $\frac{1}{x^3 + 1}$

(E)  $\frac{2x}{x^3 + 1}$

$f(x) = g(u)$  with  $u(x) = x^2$

and  $g(x) = \int_1^x \frac{1}{t^3 + 1} dt$ .

So  $f'(x) = g'(u) \frac{du}{dx}$

$$= \frac{1}{u^3 + 1} \cdot 2x$$

$$= \frac{2x}{(x^2)^3 + 1} = \frac{2x}{x^6 + 1}$$

7

$$w'(t) = \frac{\ln(t)}{t}$$

$$\int_5^{10} w'(t) dt = \int_5^{10} \frac{\ln t}{t} dt, \quad \text{let } u = \ln t$$

$$dt = \frac{1}{t} dt$$

$$t=5, u = \ln 5$$

$$t=10, u = \ln 10$$

$$= \int_{\ln 5}^{\ln 10} u \, du$$

$$= \left. \frac{u^2}{2} \right|_{\ln 5}^{\ln 10} = \frac{(\ln 10)^2}{2} - \frac{(\ln 5)^2}{2}$$

$$\approx 1.36 \text{ pounds.}$$

The integral of a rate of change gives net change  
 So this means the child gained about 1.4  
 pounds between ages 5 and 10.

8

$$\begin{aligned} \text{a) } \int_0^{\pi/4} \frac{1 + \cos^2 x}{\cos^2 x} dx &= \int_0^{\pi/4} \frac{1}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} dx \\ &= \int_0^{\pi/4} \sec^2 x + 1 \, dx \\ &= \tan x + x \Big|_0^{\pi/4} \\ &= (\tan(\pi/4) + \pi/4) - (\tan(0) + 0) \\ &= \boxed{1 + \pi/4} \end{aligned}$$

$$\begin{aligned}
 \text{b) } \int_0^1 x^{10} + 10^x dx &= \frac{x^{11}}{11} + \frac{10^x}{\ln(10)} \Big|_0^1 \\
 &= \left( \frac{1}{11} + \frac{10}{\ln(10)} \right) - \left( 0 + \frac{1}{\ln(10)} \right) \\
 &= \boxed{\frac{1}{11} + \frac{9}{\ln(10)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int \left( \frac{1+r}{r} \right)^2 dr &= \int \frac{1+2r+r^2}{r^2} dr \\
 &= \int \frac{1}{r^2} + \frac{2}{r} + 1 dr \\
 &= \boxed{-\frac{1}{r} + 2\ln|r| + r + C}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx & \quad \text{let } u = \sqrt{x} = x^{1/2} \\
 & \quad du = \frac{1}{2} x^{-1/2} dx \\
 & \quad du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2du = \frac{1}{\sqrt{x}} dx \\
 & = \int e^u \cdot 2du = 2e^u + C \\
 & = \boxed{2e^{\sqrt{x}} + C}
 \end{aligned}$$

$$e) \int_5^{10} \frac{dt}{(t-4)^2} \quad \text{let } u = t-4 \quad \begin{matrix} t=5 \Rightarrow u=1 \\ t=10 \Rightarrow u=6 \end{matrix}$$

$$du = dt$$

$$\int_1^6 \frac{1}{u^2} du = \left. -\frac{1}{u} \right|_1^6 = -\frac{1}{6} - \left(-\frac{1}{1}\right) = \boxed{\frac{5}{6}}$$

(\*) f)  
Challenge Question

$$\int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{let } u = e^x \quad du = e^x dx$$

$$\text{Note: } u^2 = e^{2x} \quad \begin{matrix} x=0 \Rightarrow u=e^0=1 \\ x=1 \Rightarrow u=e \end{matrix}$$

$$\int_1^e \frac{1}{1+u^2} du$$

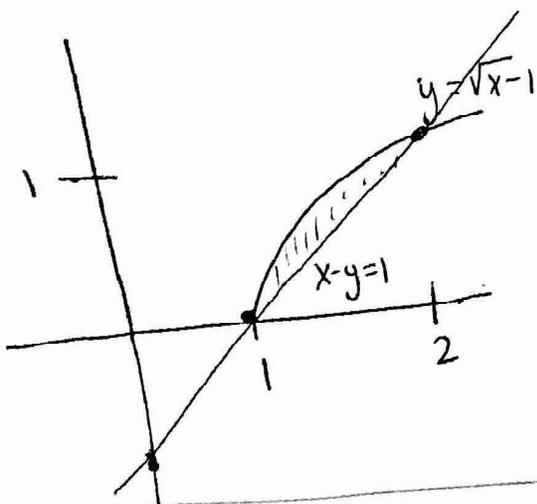
$$\text{Recall: } \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$= \arctan u \Big|_1^e$$

$$= \arctan(e) - \arctan(1)$$

$$= \boxed{\arctan(e) - \pi/4}$$

9



$$x - y = 1 \Rightarrow y = x - 1$$

$$\& \quad y = \sqrt{x - 1}$$

find intersections:

$$\text{Square both sides} \quad x - 1 = \sqrt{x - 1}$$

$$x^2 - 2x + 1 = x - 1$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

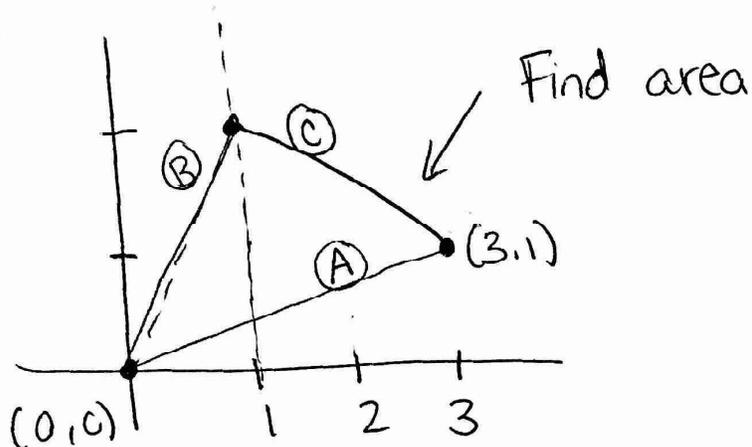
$$x = 1, 2$$

$$\text{Area: } \int_1^2 \sqrt{x-1} - (x-1) dx$$

$$u = x-1 \quad \begin{matrix} x=1, u=0 \\ x=2, u=1 \end{matrix} \quad du = dx$$

$$= \int_0^1 u^{1/2} - u du = \left. \frac{2}{3} u^{3/2} - \frac{u^2}{2} \right|_0^1 = \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6}}$$

$$= \left(-\frac{1}{4} + \frac{5}{2} - \frac{1}{6}\right)$$



$$\text{Area: } \int_0^1 \textcircled{B} - \textcircled{A} \, dx + \int_1^3 \textcircled{C} - \textcircled{A} \, dx$$

Ⓐ line ~~from~~ through (0,0) and (3,1)

$$\text{Slope: } \frac{1}{3} \quad \text{point: } (0,0)$$

$$y = \frac{1}{3}x$$

Ⓑ line ~~from~~ through (0,0) and (1,2)

$$\text{Slope: } 2 \quad \text{point } (0,0)$$

$$y = 2x$$

Ⓒ line through (1,2) & (3,1)

$$\text{Slope } \frac{2-1}{1-3} = \frac{1}{-2} = -\frac{1}{2} \quad \text{point: } (1,2)$$

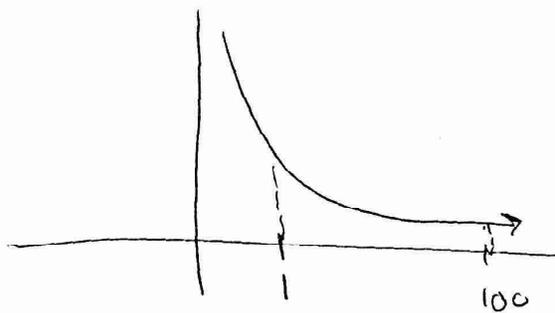
$$y - 2 = -\frac{1}{2}(x - 1) \rightarrow y = -\frac{1}{2}x + \frac{1}{2} + 2 \\ = -\frac{1}{2}x + \frac{5}{2}$$

Area:

$$\int_0^1 2x - \frac{1}{3}x \, dx + \int_1^3 \left(-\frac{1}{2}x + \frac{5}{2}\right) - \frac{1}{3}x \, dx = \boxed{\frac{5}{2}}$$

$$= x^2 - \frac{1}{6}x^2 \Big|_0^1 + \left(-\frac{1}{4}x^2 + \frac{5}{2}x - \frac{1}{6}x^2\right) \Big|_1^3 = \left(1 - \frac{1}{6}\right) + \left(-\frac{9}{4} + \frac{15}{2} - \frac{9}{6}\right) - \left(-\frac{1}{4} + \frac{5}{2} - \frac{1}{6}\right)$$

11  $y = \frac{1}{x}$



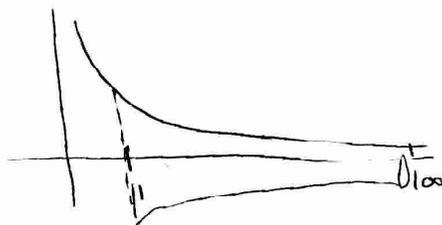
a) 
$$\int_1^{100} \frac{1}{x} dx = \ln x \Big|_1^{100} = \ln(100) - \ln(1)$$

$$= \boxed{\ln(100)}$$

b) as  $a \rightarrow \infty \int_1^a \frac{1}{x} dx = \ln x \Big|_1^a = \ln(a)$

as  $a \rightarrow \infty \ln(a) \rightarrow \infty$ , so we get infinite Area.

c) Volume:



$$\pi \int_1^{100} \left(\frac{1}{x}\right)^2 dx = \pi \int_1^{100} \frac{1}{x^2} dx$$

$$= \pi \left( -\frac{1}{x} \Big|_1^{100} \right)$$

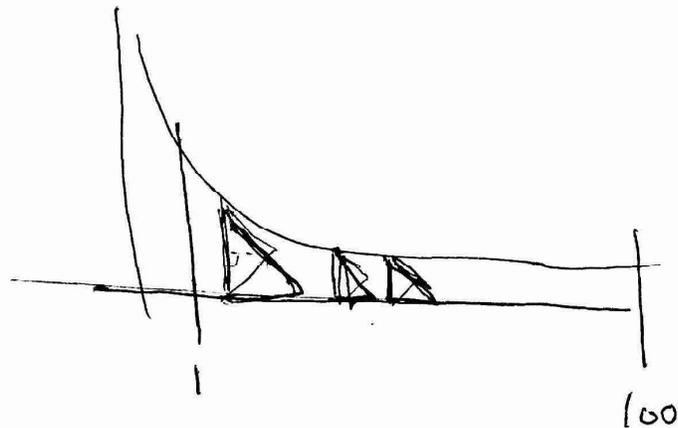
$$= \pi \left( -\frac{1}{100} + 1 \right) = \frac{99}{100} \pi$$

d) as  $a \rightarrow \infty \pi \int_1^a \left(\frac{1}{x}\right)^2 dx = \pi \left( -\frac{1}{x} \Big|_1^a \right) = \pi \left( 1 - \frac{1}{a} \right)$

So as  $a \rightarrow \infty$  Volume goes to:  $\lim_{a \rightarrow \infty} \pi \left( 1 - \frac{1}{a} \right) = \boxed{\pi}$

Volume is finite even though area is infinite!!  
 Strange but true!! More to come in Calc 2!!

e) Volume w/ cross sections  $\perp$  to x-axis  
 right triangles whose height is half  
 their base.



$$\int_1^{100} (\text{Area of Slice}) dx$$

$$= \int_1^{100} \frac{1}{2} (\text{base})(\text{height}) dx$$

base is under  $\frac{1}{x} \rightarrow \frac{1}{x}$   
 height is  $\frac{1}{2} b = \frac{1}{2}x$

$$= \int_1^{100} \frac{1}{2} \left(\frac{1}{x}\right) \left(\frac{1}{2}x\right) dx$$

$$= \int_1^{100} \frac{1}{4x^2} dx = \left. -\frac{1}{4x} \right|_1^{100} = -\frac{1}{400} + \frac{1}{4}$$

$$= \frac{1}{4} - \frac{1}{400} = \boxed{\frac{399}{400}}$$

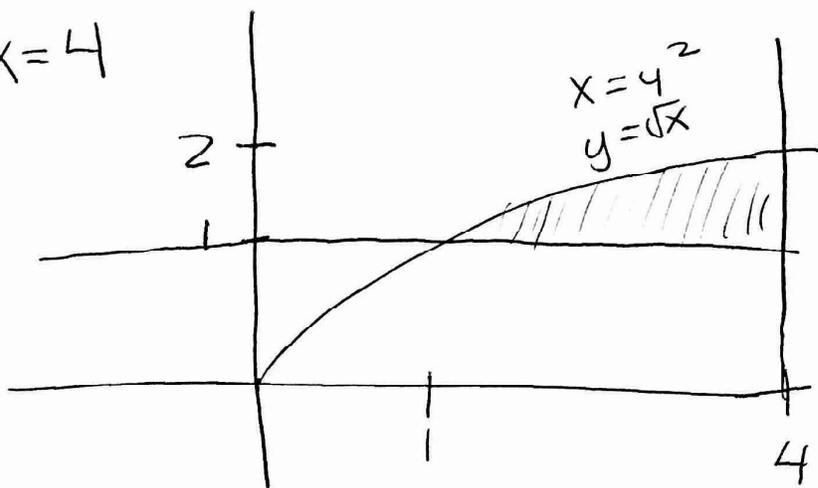
f) as  $a \rightarrow \infty$   $\int_1^a \frac{1}{4x^2} dx = \left. -\frac{1}{4x} \right|_1^a$

$$= \frac{1}{4} - \frac{1}{4a}$$

$$\lim_{a \rightarrow \infty} \frac{1}{4} - \frac{1}{4a} = \boxed{\frac{1}{4}} \quad (\text{again finite volume!})$$

12

$$y = \sqrt{x}, y = 1, x = 4$$



a) Area of region

$$\int_1^4 \sqrt{x} - 1 \, dx$$

or

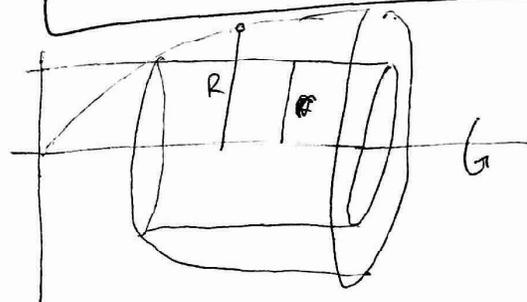
$$\int_1^2 4 - y^2 \, dy$$

b)  $\pi \int R^2 - r^2 \, dx$

$\swarrow$  outer radius     $\swarrow$  inner radius  
 $\searrow$

$$= \pi \int_1^4 (\sqrt{x})^2 - (1)^2 \, dx$$

$$= \pi \int_1^4 x - 1 \, dx$$



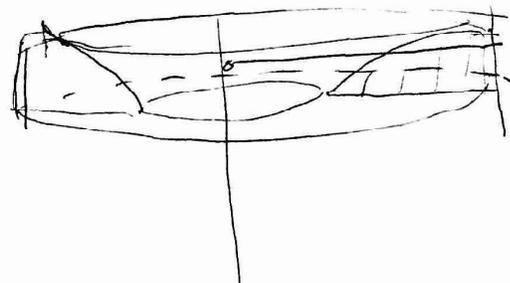
Rotate about x-axis

c) Rotate about y-axis

$$\pi \int_1^2 (\text{outer Radius})^2 - (\text{inner radius})^2 \, dy$$

$$= \pi \int_1^2 4^2 - (y^2)^2 \, dy$$

$$= \pi \int_1^2 16 - y^4 \, dy$$

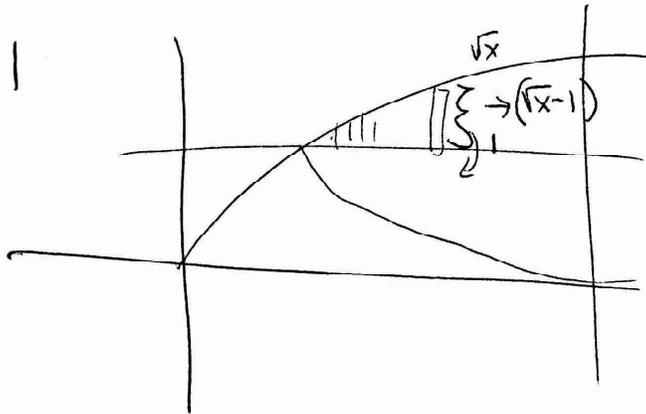


d) Rotate around  $y=1$

$$\pi \int_1^4 R^2 dx$$

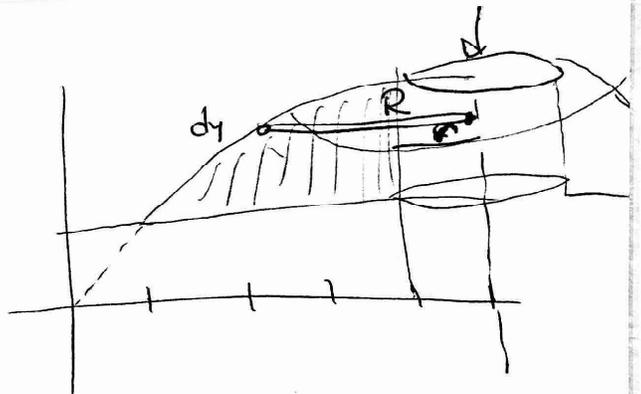
$$\pi \int_1^4 (\sqrt{x}-1)^2 dx$$

$$\text{or } \pi \int_1^4 x - 2\sqrt{x} + 1 dx$$



e) Rotate around  $x=5$

$$\pi \int_1^2 R^2 - r^2 dy$$



$R$ : outer radius from  $x = y^2$  to  $x = 5$   
Subtract right - left  $(5 - y^2)$

$r$ : inner radius from  $x = 4$  to  $x = 5$   
Subtract  $5 - 4 = 1$

$$\pi \int_1^2 (5 - y^2)^2 - 1^2 dy$$

$$\pi \int_1^2 (5 - y^2)^2 - 1 dy$$