University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 2

Sections Covered: 3.6, 3.8, 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.7, 4.8, 4.9

Read This First!

- The exam will be 50 minutes, timed, and administered via HuskyCT.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, please carefully check all submitted answers. The submitted letter answers are the **ONLY** place that counts as your official answers.
- You may use a calculator on the exam. No books or other references or are permitted, and **you are expected to work independently.**

1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

   (A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ $\boxed{\text{(B) } x_{n+1} = (x_n^2 + 7)/(2x_n)}$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

   (D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$$\left.\begin{array}{l} X_{n+1} = X_n - \dfrac{f(x_n)}{f'(x_n)} \\ f(x_n) = x_n^2 - 7 \\ f'(x_n) = 2x_n \end{array}\right\} \to X_{n+1} = X_n - \frac{x_n^2 - 7}{2x_n}$$

$$= \frac{2x_n^2 - (x_n^2 - 7)}{2x_n}$$

$$= \underline{\frac{x_n^2 + 7}{2x_n}}$$

2. Find $\dfrac{d}{dx}\left[\sin(\ln x^2)\right]$.

   (A) $\dfrac{-\cos(\ln(x))}{x^2}$ (B) $\dfrac{-2\sin(\ln(x^2))}{x^2}$ (C) $\dfrac{\cos(\ln(x))}{2x^2}$

   $\boxed{\text{(D) } \dfrac{2\cos(\ln(x^2))}{x}}$ (E) None of the above

$$\frac{d}{dx}\left[\sin(\ln x^2)\right] = \cos(\ln x^2)\cdot\frac{d}{dx}\left[\ln(x^2)\right]$$

$$= \cos(\ln x^2)\cdot\frac{1}{x^2}\cdot\frac{d}{dx}(x^2)$$

$$= \cos(\ln x^2)\cdot\frac{1}{x^2}(2x)$$

$$= \frac{2x\cos(\ln x^2)}{x^2} = \underline{\frac{2\cos(\ln x^2)}{x}}$$

3. Find $\dfrac{d}{dx}\left[\log_4(3x)\right]$.

   (A) $\dfrac{1}{3x\ln 4}$ $\boxed{\text{(B) } \dfrac{1}{x\ln 4}}$ (C) $\dfrac{1}{x}$

   (D) $\dfrac{3}{x\ln 4}$ (E) $\dfrac{3}{x}$

$$\frac{d}{dx}\left[\log_4(3x)\right] = \frac{1}{3x\ln 4}\frac{d}{dx}(3x) = \frac{3}{3x\ln 4} = \frac{1}{x\ln 4}$$

4. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

✓ I. $k > 0$

✗ ~~II.~~ $P'(5) < 0$

✓ III. $P'(10) = 100ke^{10k}$

(A) I and III only. (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

I. P increasing, so $k>0$ ✓

II. $P'(t) = kP(t) > 0$ since $k>0$ and $P>0 \rightarrow P'(5)>0$ ✗

III. $P'(t) = 100ke^{kt} \rightarrow P'(10) = 100ke^{10k}$ ✓

5. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

(D) $10e^{-t\ln(2)/20}$ (E) $10e^{t\ln(2)/20}$

$y = Ae^{kt}$, $A = 10$. At $t = 20$, $y = 5$: $5 = 10e^{k(20)}$

$\rightarrow \frac{1}{2} = e^{20k} \rightarrow 20k = \ln\frac{1}{2}$

$\rightarrow k = \frac{\ln\frac{1}{2}}{20}$

$k = \frac{-\ln 2}{20}$

$y = 10e^{-(\ln 2)t/20}$

6. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by

$(h=0)$

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ (E) $1013e^{h\ln(0.88)/1000}$

$y = Ae^{kh}$, $A = 1013$

At $t = 1000$, $y = 0.88A$

$\rightarrow 0.88(\cancel{1013}) = \cancel{1013}e^{k(1000)}$

$\rightarrow 0.88 = e^{1000k}$

$\rightarrow \ln(0.88) = 1000k \rightarrow k = \frac{\ln(0.88)}{1000}$

$\rightarrow y = 1013e^{\ln(0.88)h/1000}$

7. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

   $\boxed{\text{(A) 27 cm/s}}$ (B) 9 cm/s (C) 27/2 cm/s

   (D) 67/4 cm/s (E) None of the above

$$y^3 = x^4 + 11$$

$$\Rightarrow 3y^2 \frac{dy}{dt} = 4x^3 \frac{dx}{dt}$$

When $x = 2, y = 3, \frac{dy}{dt} = 32$:

$$3(3)^2(32) = 4(2)^3 \frac{dx}{dt}$$

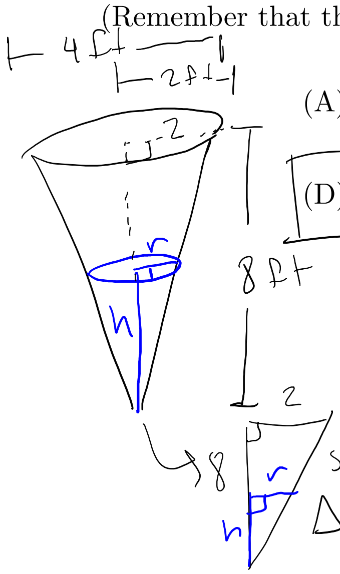
$$27 \cdot \cancel{32} = \cancel{32} \frac{dx}{dt}$$

$$\frac{dx}{dt} = 27 \text{ cm/s}$$

$\frac{dV}{dt} = -2$

8. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3} r^2 h$?)

   $h = 2$



   (A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

   $\boxed{\text{(D) } \frac{8}{\pi} \text{ ft/min}}$ (E) $\frac{16}{\pi}$ ft/min

Δs: $\frac{2}{8} = \frac{r}{h} \Rightarrow r = \frac{h}{4}$

$$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h = \frac{\pi}{3} \cdot \frac{h^3}{16} = \frac{\pi}{48} h^3 = V$$

$$\Rightarrow \frac{dV}{dt} = \frac{3\pi}{48} h^2 \frac{dh}{dt}$$

$\frac{dV}{dt} = -2, h = 2$: $-2 = \frac{\pi}{16}(2)^2 \frac{dh}{dt}$

$$\Rightarrow -2 = \frac{\pi}{4} \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{-8}{\pi}$$

9. Determine $f''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

   (A) $\frac{-1}{2x^2}$ (B) $\frac{6 \ln x}{x^4}$ (C) $\frac{1 - 6 \ln x}{x^4}$

   (D) $\frac{1 - 2 \ln x}{x^3}$ $\boxed{\text{(E) None of the above}}$

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - (\ln x)(2x)}{x^4} = \frac{x - 2x \ln x}{x^4}$$

$$f''(x) = \frac{\left[1 - \left(2\ln x + 2x \cdot \frac{1}{x}\right)\right](x^4) - 4x^3\left[x - 2x \ln x\right]}{x^8} = \frac{-1 + 6\ln x}{x^4}$$

$$= \frac{x^4\left[1 - 2\ln x + 2 - 4(1 - 2\ln x)\right]}{x^8} = \frac{3 - 4 - 2\ln x + 8\ln x}{x^4}$$

10. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ $\boxed{\text{(C) } \frac{17}{8}}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

$L(x) = f'(1)(x-1) + f(1)$

$f(1) = \sqrt{1+2+1} = 2$

$f'(x) = \frac{1}{2}(x^3 + 2x + 1)^{-1/2}(3x^2 + 2)$

$f'(1) = \frac{1}{2}(1+2+1)^{-1/2}(3+2)$

$= \frac{1}{2}\left(\frac{1}{2}\right)(5) = \frac{5}{4}$

$\to f(1.1) \approx L(1.1) = f'(1)(1.1-1) + f(1) = \frac{5}{4}(0.1) + 2$

$= \frac{1}{8} + \frac{16}{8} = \underline{\underline{\frac{17}{8}}}$

11. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 $\boxed{\text{(B) } 19/6}$ (C) 15/4

(D) 12/7 (E) 17/6

$f'(x) = 2x \to f'(3) = 6$

$f(3) = 9 - 10 = -1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-1}{6} = \frac{18}{6} + \frac{1}{6} = \underline{\underline{\frac{19}{6}}}$

12. Which of the following is the absolute maximum value of the function $f(x) = \frac{x}{x^2 + 4}$ on the interval $[0, 4]$?

(A) $\frac{1}{8}$ (B) $\frac{1}{5}$ $\boxed{\text{(C) } \frac{1}{4}}$

(D) $\frac{1}{2}$ (E) 1

$f'(x) = \frac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2}$

$= \frac{-x^2+4}{(x^2+4)^2}$

Crit #s: $x^2 = 4 \to x = \pm 2$

(denom never 0)

Only one crit # in $[0,4]$: $x = 2$

so $f(0) = 0$

$f(2) = \frac{2}{4+4} = \boxed{\frac{1}{4}} \longrightarrow$ (largest)

$f(4) = \frac{4}{16+4} = \frac{1}{5}$

13. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

(A) 9 (B) $\sqrt{27}$ $\boxed{\text{(C) } \sqrt{3}}$

(D) 3 (E) No such value of c exists.

1) f cont. on $[0,3]$ ✓

2) f diff'ble on $(0,3)$ ✓

so MVT applies ✓

$$f'(c) = \frac{f(3)-f(0)}{3-0}$$

$$3c^2 = \frac{3^3-0^3}{3-0} \to 3c^2 = \frac{27}{3} = 9 \to c^2 = 3 \to c = \pm\sqrt{3} \to c = \sqrt{3}$$

($c = -\sqrt{3}$ not in $(0,3)$)

14. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

$\boxed{\text{(A) } 1}$ (B) -2 (C) -2, 1

(D) -2, $\frac{1}{2}$ (E) -2, $\frac{1}{2}$, 1

$$f'(x) = 6x^2 + 6x - 12$$
$$= 6(x^2 + x - 2)$$
$$= 6(x-1)(x+2) = 0 \text{ (or } \cancel{\text{DNE}})$$

$x = 1, -2$ crit. #s

2nd deriv. test:

$f''(x) = 12x + 6$: $f''(1) = 18 > 0$

loc. min at $x = 1$

$f''(-2) = -18 < 0 \to$ loc. max

15. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1 $\boxed{\text{(C) } 2}$

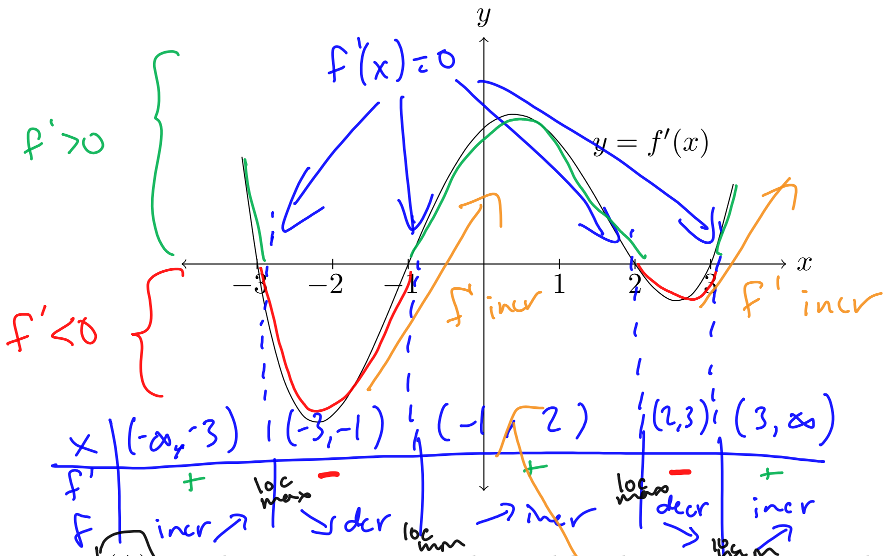
(D) 3 (E) 4

$$f'(x) = 4x^3 - 16x$$
$$f''(x) = 12x^2 - 16 = 0 \text{ (or } \cancel{\text{DNE}})$$
$$x^2 = \frac{16}{12} = \frac{4}{3}$$
$$x = \pm\frac{2}{\sqrt{3}}$$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	−	+
f	C.U.	C.D	C.U.

inflection points (2)

16. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



(A) Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

17. Referring to the same graph of the derivative in question 5, at approximately what x-value(s) is $f(x)$ concave up? $\Leftrightarrow f'' > 0 \Leftrightarrow f'$ increasing (2 intervals):

$(-2.1, 0.8)$ and $(2.6, \infty)$

(A) $x < -1$ and $x > 1.5$

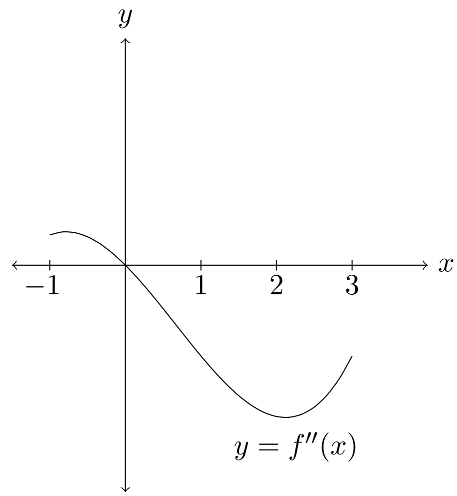
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

18. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. $f'' > 0$ here ✓

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. $f'' < 0$ here ✓

(C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to − here ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to − (f' changes ↗ to ↘) here ✓

$\boxed{\text{(E)}}$ All of the above.

19. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only $\boxed{\text{(B) } (1, 2) \text{ only}}$ (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$ (~~0, D~~NE)

$x = 2, 1$

x	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
f''	+	−	+
f	CU	CD	CU

20. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}.$$ "$=\frac{0}{0}$" $\to$ use L'Hospital's:

(A) $+\infty$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cos x}{2x}$$

$$= \text{"}\frac{1}{0}\text{"}: \frac{+}{+} \to +\infty$$

21. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}.$$ "$=\frac{0}{0}$"; use L'Hospital's:

(A) 0 (B) 1 (C) $+\infty$

(D) -1 (E) 1/2

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{L'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \underline{0}$$

22. Determine the number of inflection points of the graph of $y = x^2 - \dfrac{1}{x}$ on its domain.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$y' = 2x + \frac{1}{x^2} \to y'' = 2 - \frac{2}{x^3}$

$y''=0$ or DNE: $x=0$ or $2 = \frac{2}{x^3} \Rightarrow x^3 = 1 \to x = 1$

x	$(-\infty,0)$	$x=0$	$(0,1)$	$(1,\infty)$
y''	+	asymptote	−	+
y	$\cup$		$\cap$	$\cup$

Inflection point at $x=1$ (not at $x=0$, a vertical asymptote)

23. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 (C) 20, 40

(D) 26, 27 (E) None of the above

maximize $A = xy$ with $y + 2x = 80$: $y = 80 - 2x$, so

$A(x) = x(80-2x) = 80x - 2x^2 \to$ maximize on $(0, 40)$

$A'(x) = 80 - 4x = 0$ (or DNE)

$\to x = 20 \to y = 80 - 2x = 80 - 40$

$= 40$

2nd deriv. test:

$A''(x) = -4 < 0$ so maximum ✓

24. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) -3 (B) -2 $\boxed{\text{(C) } -1}$

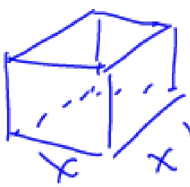
(D) 1 (E) 3

$f'(x) = 2x - \frac{3}{2}x^2 + C$, $f'(0) = C = -1 \to f'(x) = 2x - \frac{3}{2}x^2 - 1$

So $f(x) = x^2 - \frac{1}{2}x^3 - x + D$, $f(0) = D = 1 \to f(x) = x^2 - \frac{1}{2}x^3 - x + 1$

$\to f(2) = 4 - 4 - 2 + 1 = \underline{-1}$

25. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?



(A) \$500 (B) \$600 (C) \$1000

$\boxed{\text{(D) \$1200}}$ (E) \$2000

$V = x^2 y$

$V = 4000$

$\to y = \frac{4000}{x^2}$

cost = (\$1)(area of bottom + sides)

$= x^2 + 4xy \to C(x) = x^2 + 4x\left(\frac{4000}{x^2}\right) = x^2 + \frac{16000}{x}$

minimize over $(0, \infty)$.

$C'(x) = 2x - \frac{16000}{x^2} = 0$ or DNE $\to \cancel{x = 0}$, $\underline{x = 20}$

$C''(20) > 0$, so minimum ✓

$\to C(20) = \underline{\underline{1200}}$

26. Find $f(x)$ if $f'(x) = 3x^2 + \frac{2}{x}$ for $x > 0$ and $f(1) = 3$.

(A) $x^3 + 2\ln x$ (B) $x^3 - \frac{1}{x} + 3$ (C) $x^3 + 2\ln x + 1$

(D) $6x + 2\ln x - 3$ $\boxed{\text{(E) } x^3 + 2\ln x + 2}$

$f(x) = x^3 + 2\ln x + C$

$f(1) = 1 + 2\ln 1 + C = 1 + C = 3 \to C = 2$, so $\underline{f(x) = x^3 + 2\ln x + 2}$

27. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x\to\infty} \frac{f(x)}{x^2} = \text{"}\frac{?}{\infty}\text{"}$$

L'H applies if $\underline{\lim_{x\to\infty} f(x) = \infty}$

$\lim_{x\to\infty} \sin x$ DNE

$\lim_{x\to\infty} e^{-x} = 0$

$\cancel{\text{(A) } \sin(x)}$ $\cancel{\text{(B) } e^{-x}}$ $\cancel{\text{(C) } \cos(x)}$

(D) $\ln(x)$ (E) All of the above

$\lim_{x\to\infty} \ln x = \infty$ ✓

$\lim_{x\to\infty} \cos x$ DNE

cont on [0,2], so use closed interval method

28. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

(A) $1 - \ln 2$ (B) 0 $\boxed{\text{(C) } 2 - \ln 5}$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

$v'(t) = 1 - \frac{2t}{t^2+1} = 0$, ~~DNE~~ $\to t^2+1 = 2t$
$\to t^2 - 2t + 1 = 0$
$(t-1)^2 = 0$
$t = 1$
Crit # in [0,2]

$v(0) = 0 - \ln(1) = 0$ $v(2) = 2 - \ln(5) \approx 0.39$
$v(1) = 1 - \ln(2) \approx 0.31$ maximum

29. If $f(1) = 9$ and $f'(x) \ge 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

using MVT: $f'(c) = \frac{f(4) - f(1)}{4-1}$ for some c in [1,4], so

(A) 19 $\boxed{\text{(B) } 18}$ (C) 12

$3f'(c) = f(4) - f(1)$

(D) Cannot be determined (E) None of the above

$\to f(4) = 3f'(c) + f(1)$
$= 3f'(c) + 9$
$\ge 3(3) + 9 = 18$

30 Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0

min $x=-3$ CBD $x=-2$ min $x=0$ max $x=4$

crit #s: $x = -3, -2, 0, 4$ (where $f'(x) = 0$)

2nd deriv test:
$f'' > 0 \to$ loc min
$f'' < 0 \to$ loc max
$f'' = 0 \to$ CBD

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6