**University of Connecticut**
**Department of Mathematics**

Math 1131           Practice Problems for Exam 2

**Sections Covered:** 3.6, 3.8, 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.7, 4.8, 4.9

**Read This First!**

- The exam will be 50 minutes, timed, and administered via HuskyCT.

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.

- On the exam, please carefully check all submitted answers. The submitted letter answers are the **ONLY** place that counts as your official answers.

- You may use a calculator on the exam. No books or other references or are permitted, and **you are expected to work independently.**

1. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

(A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$     (B) $x_{n+1} = (x_n^2 + 7)/(2x_n)$     (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

(D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$     (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

2. Find $\dfrac{d}{dx}\left[\sin(\ln x^2)\right]$.

(A) $\dfrac{-\cos(\ln(x))}{x^2}$     (B) $\dfrac{-2\sin(\ln(x^2))}{x^2}$     (C) $\dfrac{\cos(\ln(x))}{2x^2}$

(D) $\dfrac{2\cos(\ln(x^2))}{x}$     (E) None of the above

3. Find $\dfrac{d}{dx}\left[\log_4(3x)\right]$.

(A) $\dfrac{1}{3x\ln 4}$     (B) $\dfrac{1}{x\ln 4}$     (C) $\dfrac{1}{x}$

(D) $\dfrac{3}{x\ln 4}$     (E) $\dfrac{3}{x}$

4. The size of a colony of bacteria at time $t$ hours is given by $P(t) = 100e^{kt}$, where $P$ is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

$$\text{I.} \quad k > 0$$

$$\text{II.} \quad P'(5) < 0$$

$$\text{III.} \quad P'(10) = 100ke^{10k}$$

(A) I and III only.      (B) I and II only.      (C) I only.

(D) II only.      (E) I, II, and III.

5. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time $t$ is given by

(A) $10e^{10k}$      (B) $\ln(10)e^{kt/10}$      (C) $\ln(10)e^{t/10}$

(D) $10e^{-t\ln(2)/20}$      (E) $10e^{t\ln(2)/20}$

6. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height $h$ is given by,

(A) $1000e^{10h}$      (B) $\ln(1013)e^{kh/12}$      (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$      (E) $1013e^{h\ln(0.88)/1000}$

7. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2,3)$, the $y$-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the $x$-coordinate at that instant?

(A) 27 cm/s     (B) 9 cm/s     (C) 27/2 cm/s

(D) 67/4 cm/s     (E) None of the above

8. Water is withdrawn at a constant rate of 2 ft$^3$/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height $h$ and radius $r$ is $V = \dfrac{\pi}{3}r^2h$?)

(A) $\dfrac{2}{\pi}$ ft/min     (B) $\dfrac{4}{\pi}$ ft/min     (C) $\dfrac{6}{\pi}$ ft/min

(D) $\dfrac{8}{\pi}$ ft/min     (E) $\dfrac{16}{\pi}$ ft/min

9. Determine $f''(x)$ for the function $f(x) = \dfrac{\ln x}{x^2}$.

(A) $\dfrac{-1}{2x^2}$     (B) $\dfrac{6\ln x}{x^4}$     (C) $\dfrac{1 - 6\ln x}{x^4}$

(D) $\dfrac{1 - 2\ln x}{x^3}$     (E) None of the above

10. Use the linearization for the function $f(x) = \sqrt{x^3 + 2x + 1}$ at $x = 1$ to approximate the value of $f(1.1)$.

$$\text{(A)} \ \frac{161}{80} \qquad \text{(B)} \ \frac{21}{10} \qquad \text{(C)} \ \frac{17}{8}$$

$$\text{(D)} \ \frac{1}{2} \qquad \text{(E)} \ \frac{17}{16}$$

11. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine $x_2$.

$$\text{(A)} \ 1/2 \qquad \text{(B)} \ 19/6 \qquad \text{(C)} \ 15/4$$

$$\text{(D)} \ 12/7 \qquad \text{(E)} \ 17/6$$

12. Which of the following is the absolute maximum value of the function $f(x) = \dfrac{x}{x^2 + 4}$ on the interval $[0, 4]$?

$$\text{(A)} \ \frac{1}{8} \qquad \text{(B)} \ \frac{1}{5} \qquad \text{(C)} \ \frac{1}{4}$$

$$\text{(D)} \ \frac{1}{2} \qquad \text{(E)} \ 1$$

13. Find all value(s) of the number $c$ that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

$$\text{(A) } 9 \qquad \text{(B) } \sqrt{27} \qquad \text{(C) } \sqrt{3}$$

$$\text{(D) } 3 \qquad \text{(E) No such value of } c \text{ exists.}$$

14. Find all value(s) of $x$ where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

$$\text{(A) } 1 \qquad \text{(B) } -2 \qquad \text{(C) } -2,\ 1$$

$$\text{(D) } -2,\ \frac{1}{2} \qquad \text{(E) } -2,\ \frac{1}{2},\ 1$$

15. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

$$\text{(A) } 0 \qquad \text{(B) } 1 \qquad \text{(C) } 2$$

$$\text{(D) } 3 \qquad \text{(E) } 4$$

16. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what $x$-value(s) does $f(x)$ have a local maximum or local minimum?



$$y$$

$$y = f'(x)$$

$$x$$

$-3 \quad -2 \quad -1 \qquad 1 \quad 2 \quad 3$
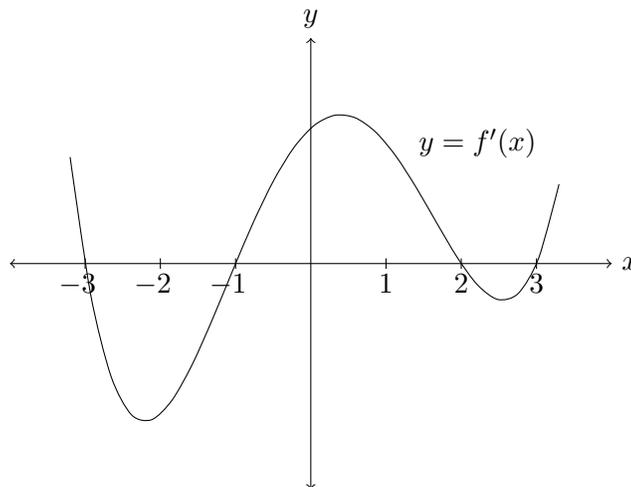
(A) Local maxima at $-3$ and $2$ and local minima at $-1$ and $3$

(B) Local maxima at $-1$ and $3$ and local minima at $-3$ and $2$

(C) Local maxima at $-1$ and $3$ and local minimum at $2$

(D) Local maxima at $-3$ and $2$ and local minimum at $-1$

(E) None of the above

17. Referring to the same graph of the derivative in question 16, at approximately what $x$-value(s) is $f(x)$ concave up?

(A) $x < -1$ and $x > 1.5$

(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

18. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



$$y = f''(x)$$
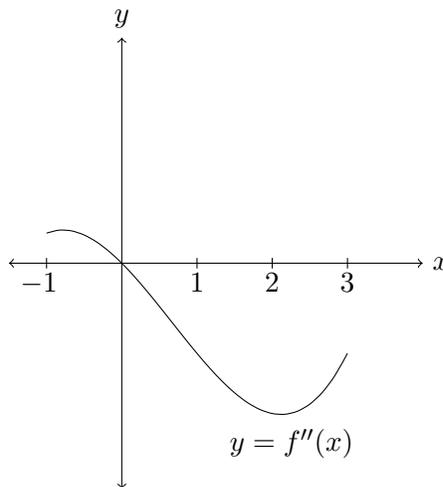
    (A) The function $f(x)$ is concave up when $-1 < x < 0$.

    (B) The derivative $f'(x)$ is decreasing when $0 < x < 3$.

    (C) The function $f(x)$ has a point of inflection at $x = 0$.

    (D) The derivative $f'(x)$ has a local maximum at $x = 0$.

    (E) All of the above.

19. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

    (A) $(-\infty, 1)$ only    (B) $(1, 2)$ only    (C) $(-\infty, -1)$ and $(2, \infty)$

    (D) $(2, \infty)$ only    (E) $(-\infty, 1)$ and $(2, \infty)$

20. Evaluate the following limit:
$$\lim_{x \to 0^+} \frac{\sin x}{x^2}.$$

      (A) $+\infty$     (B) $-\infty$     (C) 0

      (D) 1/2     (E) $-1/2$

21. Evaluate the following limit:
$$\lim_{x \to \pi/2} \frac{1 - \sin x}{\cos x}.$$

      (A) 0     (B) 1     (C) $+\infty$

      (D) $-1$     (E) 1/2

22. Determine the number of inflection points of the graph of $y = x^2 - \dfrac{1}{x}$ on its domain.

      (A) 0     (B) 1     (C) 2     (D) 3     (E) 4

23. Find two positive numbers $x$ and $y$ satisfying $y + 2x = 80$ whose product is a maximum.

    (A) 24, 32      (B) 26, 28      (C) 20, 40

    (D) 26, 27      (E) None of the above

24. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

    (A) $-3$      (B) $-2$      (C) $-1$

    (D) 1      (E) 3

25. A box with square base and open top must have a volume of 4000 cm$^3$. If the cost of the material used is \$1/cm$^2$, then what is the smallest possible cost of the box?

    (A) \$500      (B) \$600      (C) \$1000

    (D) \$1200      (E) \$2000

26. Find $f(x)$ if $f'(x) = 3x^2 + \dfrac{2}{x}$ for $x > 0$ and $f(1) = 3$.

   (A) $x^3 + 2\ln x$      (B) $x^3 - \dfrac{1}{x} + 3$      (C) $x^3 + 2\ln x + 1$

   (D) $6x + 2\ln x - 3$      (E) $x^3 + 2\ln x + 2$

27. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$$\lim_{x \to \infty} \frac{f(x)}{x^2}$$

   (A) $\sin(x)$      (B) $e^{-x}$      (C) $\cos(x)$

   (D) $\ln(x)$      (E) All of the above

28. A particle moves along a line with velocity $v(t) = t - \ln(t^2 + 1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

   (A) $1 - \ln 2$      (B) $0$      (C) $2 - \ln 5$

   (D) $\ln 2 - 1$      (E) $\ln 5 - 2$

29. If $f(1) = 9$ and $f'(x) \geq 3$ for all $x$ in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

    (A) 19      (B) 18      (C) 12

    (D) Cannot be determined      (E) None of the above

30. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

| $x$ | $-7$ | $-3$ | $-2$ | $0$ | $1$ | $4$ | $6$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $0$ | $0$ | $3$ | $-10$ | $0$ | $25$ | $2$ |
| $f'(x)$ | $-4$ | $0$ | $0$ | $0$ | $9$ | $0$ | $2$ |
| $f''(x)$ | $5$ | $1$ | $0$ | $8$ | $-7$ | $-3$ | $0$ |

    (A) Local max at 1 and 4; local min at $-7$, $-3$, and 0; CBD at $-2$ and 6

    (B) Local max at $-3$ and 0; local min at 4; CBD at $-2$

    (C) Local max at 4; local min at $-3$ and 0; CBD at $-2$

    (D) Local max at 4; local min at 0

    (E) Local max at $-7$, $-3$, and 0; local min at 1 and 4; CBD at $-2$ and 6