Math 1131 Week 7 Worksheet

Name: ________________________

Discussion Section:_____________

Solutions should show all of your work, not just a single final answer.

3.8: Exponential Growth and Decay

1. In 1859, 24 rabbits were released into the wild in Australia, where they had no natural predators. Their population grew exponentially, doubling every 6 months.

   (a) Determine $P(t)$, the function that gives the population at time t, **and** the differential equation describing the population growth. Let units for t be years since 1859.

   (b) After how many years, rounded to one digit after the decimal point, did the rabbit population reach 1,000,000?

   (c) Determine the *rate* of population change, in rabbits/year, midway through the third year. (**Warning**: t is not 3.5, just like the year midway through the 21st century is not 2150.) Round the final answer to 2 digits after the decimal point.

2. The element Unobtainium has a half-life of 3 years. Let $M(t)$ be the mass of Unobtainium at time t starting with an initial amount of 14 kg.

   (a) Give a formula for $M(t)$.

   (b) After how many years will the initial mass of Unobtainium shrink to 1 kg? Round your answer to one digit after the decimal point.

3. Starbucks serves coffee at $170°$ and the room temperature in Starbucks is $70°$. The coffee cools to $100°$ after 10 minutes. Let $T(t)$ be the temperature of the coffee at time t, measured in minutes.

   (a) Write down the differential equation for $T(t)$ and determine a formula for $T(t)$.

   (b) From the time when the temperature is $100°$ at $t = 10$, how many *additional* minutes will it take for the temperature of the coffee to reach $80°$? Round your answer to one digit after the decimal point.

4. T/F (with justification) If $\dfrac{dy}{dx} = y$ then $y = 0$ or $y = e^x$.

5. T/F (with justification) A function $y(t)$ satisfying $\dfrac{dy}{dt} = -.01y$ has $\lim_{t\to\infty} y(t) = 0$.

3.9: Related Rates

6. The radius r of a spherical balloon is expanding at the constant rate of 14 in/min.

   (a) Determine the rate at which the volume V changes with respect to time, in in^3/min, when $r = 8$ in. Round your answer to the nearest integer. Recall $V = \frac{4}{3}\pi r^3$.

   (b) Determine the rate at which the surface area S changes with respect to time, in in^2/min, when $r = 8$ in. Round your answer to the nearest integer. Recall $S = 4\pi r^2$.

   (c) If the radius doubles, does dV/dt double? Does dS/dt double?

7. Water is flowing into an upside-down right circular cone with height 3 m and radius 2 m at the top. As water fills the cone, let the height of the water in the cone be h m, and let r m be the radius of the top of the water.

   (a) Draw and label a diagram of this scenario, find an expression for r in terms of h, then find an expression for the volume V of water in the cone in terms of h alone (no r in the formula). Recall the volume of a right-circular cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.

   (b) Express dV/dt in terms of h and dh/dt. If dV/dt is constant (and not zero), explain from your formula why dh/dt cannot be constant as well.

   (c) Assuming the water flows into the cone at a constant rate of 2 m^3/min, how quickly is its height changing, in m/min, when the height is 2 m? Round your answer to the nearest tenth.

8. A cop sits in a parked car 10 feet from a straight road. As you drive along the road, the cop aims a radar gun at your car. Let s be the distance from your car to the cop in feet. The radar gun measures the rate at which your distance from the cop is changing with respect to time, which is ds/dt. Let x be the distance, in feet, of your car from the point on the road that is closest to the cop, so your car's velocity is dx/dt.

   (a) Draw and label a diagram of this scenario, and write $\dfrac{dx}{dt}$ in terms of $\dfrac{ds}{dt}$, s, and x.

   (b) Use part a to explain why $\dfrac{ds}{dt} < \dfrac{dx}{dt}$. Thus the radar gun's measurement of $\dfrac{ds}{dt}$ always *underestimates* your car's velocity. (This is why if the radar gun measures a speed greater than the speed limit on the road, the driver deserves a ticket.)

   (c) Does the conclusion in part (b) depend on the cop's car being 10 feet, rather than some other (positive) distance, from the road?

3.10: Linear Approximations and Differentials

9. (a) Find the linearization of the function $f(x) = \sqrt{x}$ at 9.

   (b) Use the linear approximation obtained in part (a) (no other methods) to approximate $\sqrt{9.2}$. Your answer based on that linearization can be given either as an exact fraction or rounded to four digits after the decimal point.

10. (a) Find the linearization of the function $f(x) = \dfrac{1}{1+x^2}$ at 7.

    (b) Use the linear approximation obtained in part (a) (no other methods) to approximate $\dfrac{1}{37}$. Round your answer to three digits after the decimal point.

11. The side length of a cube is measured to be $x = 1.3$ feet, with an error of at most 1 inch.

(a) (No calculus) Determine the difference, in ft^3, between the volume of the cube computed with the measured side length and the volume computed with the largest (resp., smallest) value for the side length in the error range. Remember first to convert all lengths to feet! Round your final answers to three digits after the decimal point.

(b) Use differentials to estimate the error in calculating the volume of the cube using the measured value and error estimate for the side length of the cube. Round your final answer to three digits after the decimal point.