Math 1131 Week 13 Worksheet

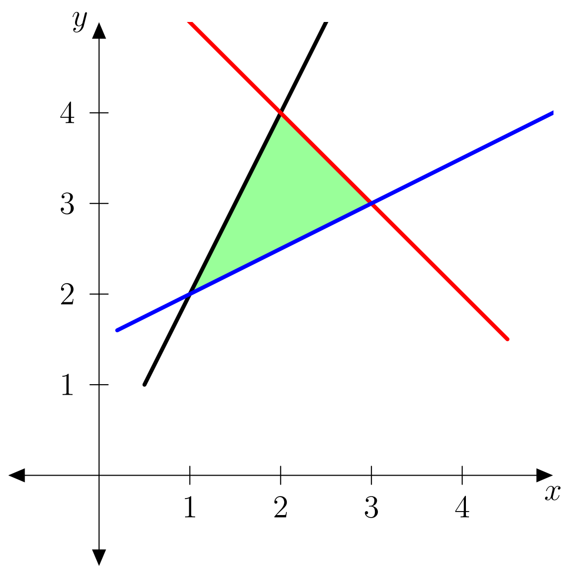
Name: ________________________

Discussion Section:______________

Solutions should show all of your work, not just a single final answer.

6.1: Areas Between Curves

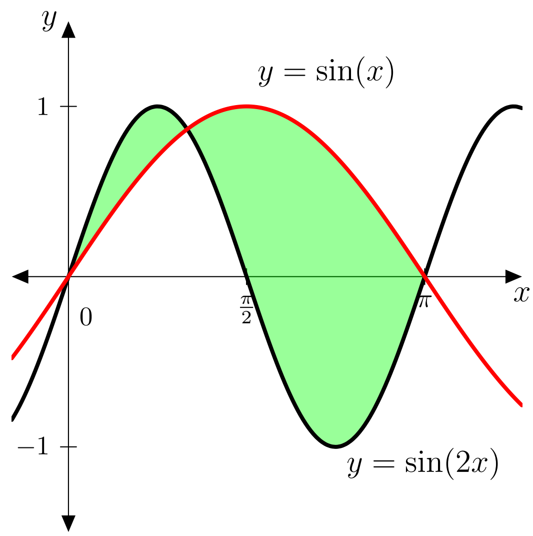
1. We want to find the area of the triangle with vertices $(1, 2)$, $(2, 4)$, and $(3, 3)$, by calculus.



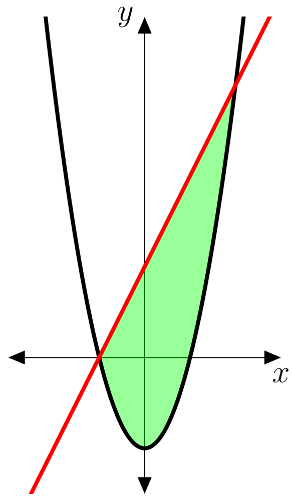
   (a) Find equations for the three lines connecting the vertices.

   (b) Use the equations in part (a) to express the area of the triangle in terms of integrals, and then compute the area.

2. Find the area of the regions below, between $y = \sin x$ and $y = \sin(2x)$ for $0 \leq x \leq \pi$. (Hint: To find the exact coordinates of the point where the graphs cross, recall that $\sin(2x) = 2\sin x \cos x$.)



3. We want to find the area of the region bounded by $y = 2x + 4$ and $y = x^2 - 4$.



(a) Determine the coordinates of the points where the line and parabola intersect.

(b) Express the area as an integral with respect to x.

(c) Express the area as an integral with respect to y.

(d) Explain which of (a) or (b) is simpler to compute, and use the simpler one to find the area. Simplify your final answer.

6.2: Volumes

4. Set up, but **do not evaluate**, a definite integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

   (a) $y = 3 - x$, $y = 0$, $x = 1$, $x = 2$; about the x-axis

   (b) $y = x^4 - 2x^2 + 1$, $y = 2 - 2x^2$; about the x-axis

   (c) $y^2 = x$, $x = 2y$; about the y-axis

5. A solid region has a circular base of radius 3 whose cross-sections perpendicular to the x-axis are equilateral triangles.

   (a) Placing the circular base in the plane so it's centered at the origin, determine the side length of the cross-sectional triangle that passes through $(x, 0)$, for $-3 \leq x \leq 3$. (Your final answer will depend on x.) Draw a clear diagram in your solution.

   (b) Set up, but **do not evaluate**, an integral equal to the volume of this solid region. Hint: the area of an equilateral triangle with side length s is $\frac{s^2}{4}\sqrt{3}$.