Math 1131 Week 12 Worksheet

Name: ________________________

Discussion Section:______________

Solutions should show all of your work, not just a single final answer.

5.3: The Fundamental Theorem of Calculus

1. In worksheet 5.1 you used rectangles to estimate the area under the curve $y = 4 - x^2$ over the interval $[0, 2]$. Use the Fundamental Theorem of Calculus to compute the exact area and then determine if the midpoint approximation from problem 1c in worksheet 5.1 is an underestimate or an overestimate of that area.

2. Use Part 1 of the Fundamental Theorem of Calculus to compute $f'(x)$ in each case below.

   (a) $f(x) = \displaystyle\int_1^x \frac{1}{t^4+1}\, dt$

   (b) $f(x) = \displaystyle\int_x^1 \cos\sqrt{t}\, dt$

   (c) $f(x) = \displaystyle\int_1^{3x} \ln t\, dt$

   (d) $f(x) = \displaystyle\int_0^{x^2} t\sin t\, dt$

3. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_2^3 (x^3 + x)\ dx$

(b) $\displaystyle\int_{-1}^1 x^{20}\ dx$

(c) $\displaystyle\int_2^{10} \frac{1}{\sqrt{x}}\ dx$

(d) $\displaystyle\int_0^1 e^{x+1}\ dx$

4. (a) Let $A_0(x) = \int_0^x (1-t^2)\,dt$, $A_1(x) = \int_1^x (1-t^2)\,dt$, and $A_2(x) = \int_2^x (1-t^2)\,dt$. Compute these explicitly in terms of x using Part 2 of the Fundamental Theorem of Calculus.

(b) Over the interval $[0, 2]$, use your answers in part (a) to sketch the graphs of $y = A_0(x)$, $y = A_1(x)$, and $y = A_2(x)$ on the same set of axes.

(c) How are the three graphs in part (a) related to each other? In particular, what does Part 1 of the Fundamental Theorem of Calculus tell you about the graphs in part (a)?

(d) On a graph of $y = 1 - t^2$, for $0 \leq t \leq 2$, shade the region with signed area $A_0(1.5)$. Indicate with $+$ and $-$ which area counts positively and which negatively.

5. T/F (with justification) The function $F(x) = \int_0^x \cos(t^2)\,dt$ is an antiderivative of $\cos(x^2)$.

6. T/F (with justification) $\displaystyle\int_{-2}^{2} x^{-4}\,dx = \left.\frac{x^{-3}}{-3}\right|_{-2}^{2} = -\frac{1}{12}$.

5.4: Indefinite Integrals and the Net Change Theorem

7. Evaluate each indefinite integral.

   (a) $\int \left(x^2 + \frac{1}{x^3} \right) \, dx$

   (b) $\int \frac{x^3 - 2\sqrt{x}}{x} \, dx$

   (c) $\int \frac{1}{\cos^2 x} dx \quad$ (Hint: $\frac{1}{\cos x} = \sec x$)

8. Water is released into a tank at the rate $r(t) = 5 + \sqrt{t}$ ft^3/min at time t (in minutes). At time $t = 1$, there is 12 ft^3 of water in the tank.

(a) Evaluate $\int_1^6 r(t)\,dt$, rounding your answer to two decimal places.

(b) In the context given above, what does the value in part (a) tell us?

(c) Determine the volume of water in the tank at time $t = 6$.

9. T/F (with justification) $\int \cos(x^2)\,dx = \sin(x^2) + C$.

5.5: The Substitution Rule

10. Evaluate each of the following indefinite integrals using substitution, expressing your final answer in terms of x.

    (a) $\int x^2 \sin(x^3)\, dx$

    (b) $\int x\sqrt{4x+1}\, dx$

    (c) $\int \frac{x}{x^2+1}\, dx$

    (d) $\int \frac{1}{x \ln x}\, dx$

11. Rewrite each of the following definite integrals in x as a definite integral in the indicated new variable u. **Do not evaluate** the new definite integral.

(a) $\int_0^1 x^2(1+2x^3)^5\,dx$ in terms of $u = 1+2x^3$

(b) $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$ in terms of $u = \cos x$

(c) $\int_0^{\pi/3} \sin x \cos x\,dx$ in terms of $u = \cos x$

(d) $\int_2^3 xe^{-x^2}\,dx$ in terms of $u = x^2$

12. (T/F) When $u = \sqrt{x}$, $\int_0^4 f(\sqrt{x})\,dx = \int_0^2 2uf(u)\,du$.