

1. Review Practice Midterms 1-3!

2. $w'(t) = \frac{\ln(t)}{t}$

$$\int_5^{10} w'(t) dt = \int_5^{10} \frac{\ln t}{t} dt, \quad \text{let } u = \ln t$$

$$dt = \frac{1}{t} dt$$

$$t=5, u = \ln 5$$

$$t=10, u = \ln 10$$

$$= \int_{\ln 5}^{\ln 10} u \, du$$

$$= \frac{u^2}{2} \Big|_{\ln 5}^{\ln 10} = \frac{(\ln(10))^2}{2} - \frac{(\ln(5))^2}{2}$$

$$\approx 1.36 \text{ pounds.}$$

The integral of a rate of change gives net change
So this means the child gained about 1.4
pounds between ages 5 and 10.

3. a) $\int_0^{\pi/4} \frac{1 + \cos^2 x}{\cos^2 x} dx = \int_0^{\pi/4} \frac{1}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} dx$

$$= \int_0^{\pi/4} \sec^2 x + 1 \, dx$$
$$= \tan x + x \Big|_0^{\pi/4}$$
$$= (\tan(\pi/4) + \pi/4) - (\tan(0) + 0)$$
$$= \boxed{1 + \pi/4}$$

$$\begin{aligned}
 \text{b) } \int_0^1 x^{10} + 10^x dx &= \frac{x^{11}}{11} + \frac{10^x}{\ln(10)} \Big|_0^1 \\
 &= \left(\frac{1}{11} + \frac{10}{\ln(10)} \right) - \left(0 + \frac{1}{\ln(10)} \right) \\
 &= \boxed{\frac{1}{11} + \frac{9}{\ln(10)}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } \int \left(\frac{1+r}{r} \right)^2 dr &= \int \frac{1+2r+r^2}{r^2} dr \\
 &= \int \frac{1}{r^2} + \frac{2}{r} + 1 dr \\
 &= \boxed{-\frac{1}{r} + 2\ln|r| + r + C}
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx & \quad \text{let } u = \sqrt{x} = x^{1/2} \\
 & \quad du = \frac{1}{2} x^{-1/2} dx \\
 & \quad du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2du = \frac{1}{\sqrt{x}} dx \\
 & = \int e^u \cdot 2du = 2e^u + C \\
 & = \boxed{2e^{\sqrt{x}} + C}
 \end{aligned}$$

$$e) \int_5^{10} \frac{dt}{(t-4)^2} \quad \text{let } u = t-4 \quad \begin{matrix} t=5 \Rightarrow u=1 \\ t=10 \Rightarrow u=6 \end{matrix}$$

$$du = dt$$

$$\int_1^6 \frac{1}{u^2} du = \left. -\frac{1}{u} \right|_1^6 = -\frac{1}{6} - \left(-\frac{1}{1}\right)$$

$$= \boxed{\frac{5}{6}}$$

(*) f)
Challenge
Question

$$\int_0^1 \frac{e^x}{1+e^{2x}} dx$$

$$\text{let } u = e^x \quad du = e^x dx$$

$$\text{Note: } u^2 = e^{2x} \quad \begin{matrix} x=0 \Rightarrow u=e^0=1 \\ x=1 \Rightarrow u=e \end{matrix}$$

$$\int_1^e \frac{1}{1+u^2} du$$

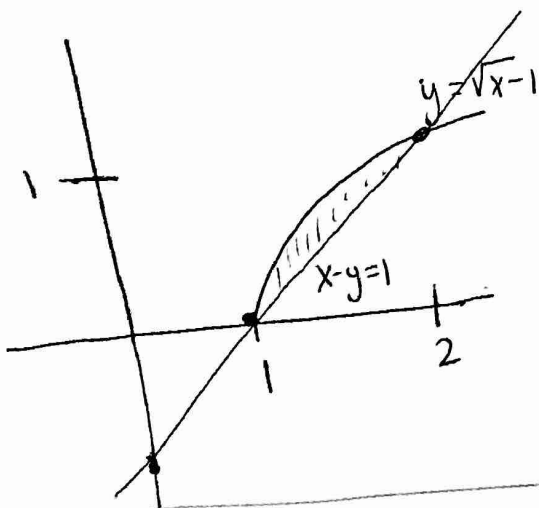
$$\text{Recall: } \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$= \arctan u \Big|_1^e$$

$$= \arctan(e) - \arctan(1)$$

$$= \boxed{\arctan(e) - \pi/4}$$

4.



$$x - y = 1 \Rightarrow y = x - 1$$

$$\& \quad y = \sqrt{x - 1}$$

find intersections:

$$\text{Square both sides} \quad x - 1 = \sqrt{x - 1}$$

$$x^2 - 2x + 1 = x - 1$$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$x = 1, 2$$

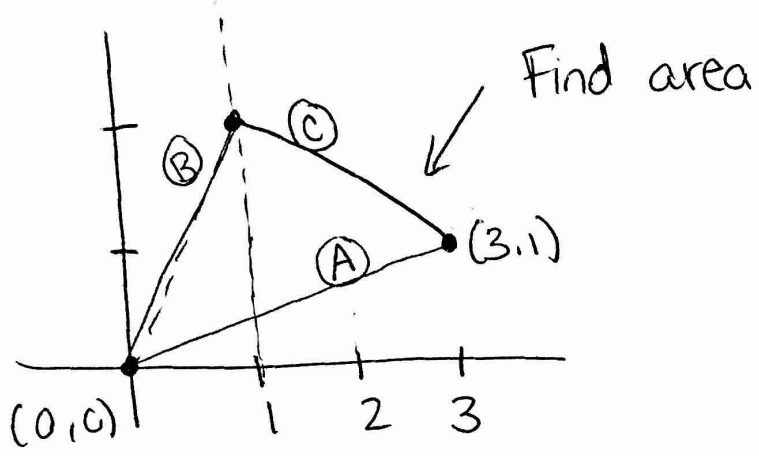
$$\text{Area: } \int_1^2 \sqrt{x-1} - (x-1) dx$$

$$= \int_0^1 u^{1/2} - u du \quad \begin{matrix} u = x-1 & x=1, u=0 \\ & x=2, u=1 \end{matrix} \quad du = dx$$

$$= \left. \frac{2}{3} u^{3/2} - \frac{u^2}{2} \right|_0^1 = \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6}}$$

$$= \left(-\frac{1}{4} + \frac{5}{2} - \frac{1}{6}\right)$$

5.



$$\text{Area: } \int_0^1 \textcircled{B} - \textcircled{A} \, dx + \int_1^3 \textcircled{C} - \textcircled{A} \, dx$$

Ⓐ line ~~from~~ through (0,0) and (3,1)

Slope: $\frac{1}{3}$ point: (0,0)

$$y = \frac{1}{3}x$$

Ⓑ line ~~from~~ through (0,0) and (1,2)

Slope: 2 point (0,0)

$$y = 2x$$

Ⓒ line through (1,2) & (3,1)

Slope $\frac{2-1}{1-3} = \frac{1}{-2} = -\frac{1}{2}$ point: (1,2)

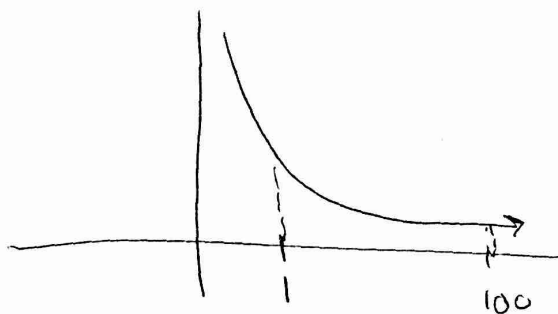
$$y - 2 = -\frac{1}{2}(x - 1) \rightarrow y = -\frac{1}{2}x + \frac{1}{2} + 2 = -\frac{1}{2}x + \frac{5}{2}$$

Area:

$$\int_0^1 2x - \frac{1}{3}x \, dx + \int_1^3 \left(-\frac{1}{2}x + \frac{5}{2}\right) - \frac{1}{3}x \, dx = \boxed{\frac{5}{2}}$$

$$= x^2 - \frac{1}{6}x^2 \Big|_0^1 + \left(-\frac{1}{4}x^2 + \frac{5}{2}x - \frac{1}{6}x^2\right) \Big|_1^3 = \left(1 - \frac{1}{6}\right) + \left(-\frac{9}{4} + \frac{15}{2} - \frac{9}{6}\right) - \left(-\frac{1}{4} + \frac{5}{2} - \frac{1}{6}\right)$$

b. $y = \frac{1}{x}$



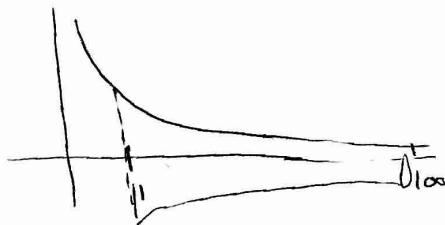
a)
$$\int_1^{100} \frac{1}{x} dx = \ln x \Big|_1^{100} = \ln(100) - \ln(1)$$

$$= \boxed{\ln(100)}$$

b) as $a \rightarrow \infty \int_1^a \frac{1}{x} dx = \ln x \Big|_1^a = \ln(a)$

as $a \rightarrow \infty \ln(a) \rightarrow \infty$, so we get infinite Area.

c) Volume:



$$\pi \int_1^{100} \left(\frac{1}{x}\right)^2 dx = \pi \int_1^{100} \frac{1}{x^2} dx$$

$$= \pi \left(-\frac{1}{x} \Big|_1^{100} \right)$$

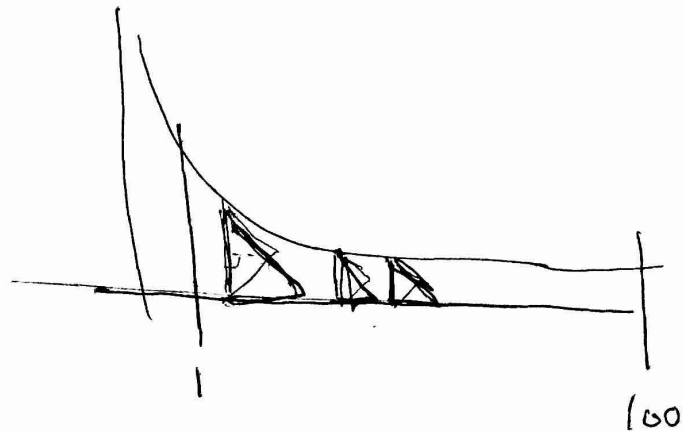
$$= \pi \left(-\frac{1}{100} + 1 \right) = \frac{99}{100} \pi$$

d) as $a \rightarrow \infty \pi \int_1^a \left(\frac{1}{x}\right)^2 dx = \pi \left(-\frac{1}{x} \Big|_1^a \right) = \pi \left(1 - \frac{1}{a} \right)$

So as $a \rightarrow \infty$ Volume goes to: $\lim_{a \rightarrow \infty} \pi \left(1 - \frac{1}{a} \right) = \boxed{\pi}$

Volume is finite even though area is infinite!!
 Strange but true!! More to come in Calc 2!!

e) Volume w/ cross sections \perp to x-axis
 right triangles whose height is half
 their base.



$$\int_1^{100} (\text{Area of Slice}) dx$$

$$= \int_1^{100} \frac{1}{2} (\text{base})(\text{height}) dx$$

base is under $1/x \Rightarrow 1/x$
 height is $\frac{1}{2} b = \frac{1}{2}x$

$$= \int_1^{100} \frac{1}{2} \left(\frac{1}{x}\right) \left(\frac{1}{2}x\right) dx$$

$$= \int_1^{100} \frac{1}{4x^2} dx = \left. -\frac{1}{4x} \right|_1^{100} = -\frac{1}{400} + \frac{1}{4}$$

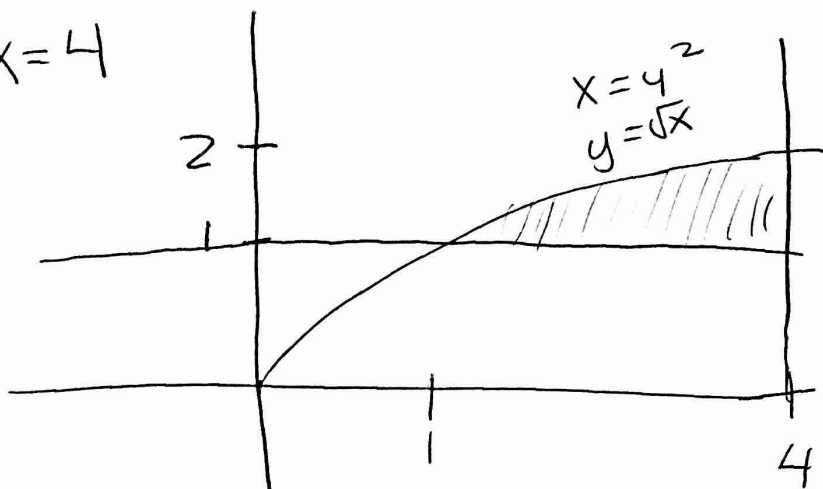
$$= \frac{1}{4} - \frac{1}{400} = \boxed{\frac{399}{400}}$$

f) as $a \rightarrow \infty$ $\int_1^a \frac{1}{4x^2} dx = \left. -\frac{1}{4x} \right|_1^a$

$$= \frac{1}{4} - \frac{1}{4a}$$

$$\lim_{a \rightarrow \infty} \left(\frac{1}{4} - \frac{1}{4a} \right) = \boxed{\frac{1}{4}} \quad (\text{again finite volume!})$$

7. $y = \sqrt{x}$, $y = 1$, $x = 4$



a) Area of region

$$\int_1^4 \sqrt{x} - 1 \, dx$$

or

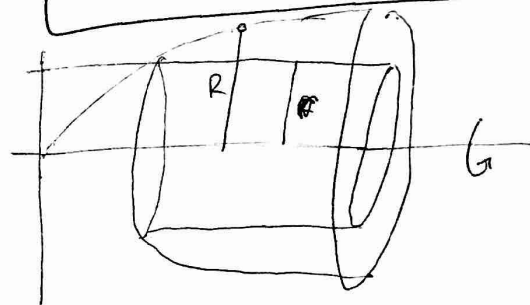
$$\int_1^2 4 - y^2 \, dy$$

b) $\pi \int R^2 - r^2 \, dx$

\swarrow outer radius \swarrow inner radius

$$= \pi \int_1^4 (\sqrt{x})^2 - (1)^2 \, dx$$

$$= \pi \int_1^4 x - 1 \, dx$$



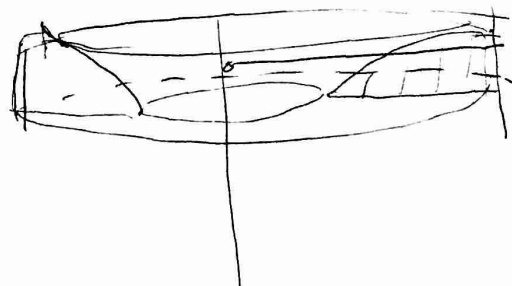
Rotate about x-axis

c) Rotate about y-axis

$$\pi \int_1^2 (\text{outer Radius})^2 - (\text{inner radius})^2 \, dy$$

$$= \pi \int_1^2 4^2 - (y^2)^2 \, dy$$

$$= \pi \int_1^2 16 - y^4 \, dy$$

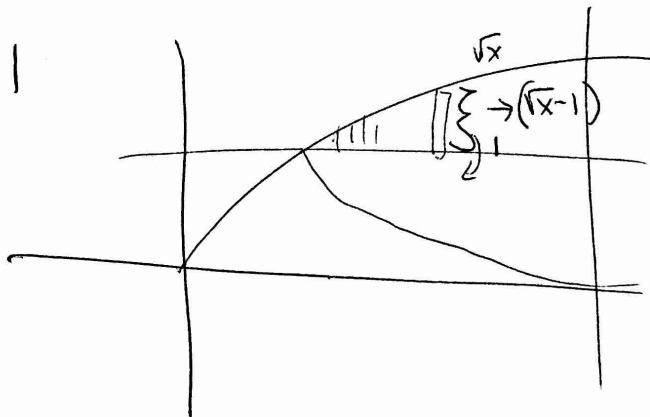


d) Rotate around $y=1$

$$\pi \int_1^4 R^2 dx$$

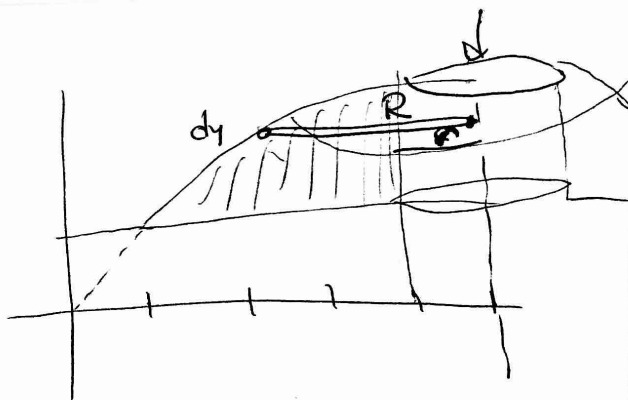
$$\pi \int_1^4 (\sqrt{x}-1)^2 dx$$

$$\text{or } \pi \int_1^4 x - 2\sqrt{x} + 1 dx$$



e) Rotate around $x=5$

$$\pi \int_1^2 R^2 - r^2 dy$$



R : outer radius from $x = y^2$ to $x = 5$
Subtract right - left $(5 - y^2)$

r : inner radius from $x = 4$ to $x = 5$
Subtract $5 - 4 = 1$

$$\pi \int_1^2 (5 - y^2)^2 - 1^2 dy$$

$$= \pi \int_1^2 (5 - y^2)^2 - 1 dy$$