University of Connecticut
Department of Mathematics

MATH 1131 EXAM 3 PRACTICE PROBLEMS

Solutions

Sections Covered: 4.1, 4.2, 4.3, 4.4, 4.7, 4.9, 5.1, 5.2, 5.3

Read This First!

- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- The exam will be 50 minutes, timed, and administered via HuskyCT.
- You are not required to use Lockdown Browser, ProctorU, or similar video proctoring. Instead, questions will be aimed more toward measuring your conceptual understanding of the topics rather than your ability to compute derivatives, antiderivatives, integrals, limits, etc.
- Calculators, books, and notes are allowed on the exam, but **you are expected to work independently.**

1. Which of the following is the absolute maximum value of the function $f(x) = \dfrac{x}{x^2+4}$ on the interval $[0, 4]$?

   (A) $\frac{1}{8}$ (B) $\frac{1}{5}$ $\boxed{\text{(C) } \frac{1}{4}}$

   (D) $\frac{1}{2}$ (E) 1

$f'(x) = \dfrac{(x^2+4)1 - x(2x)}{(x^2+4)^2} = \dfrac{-x^2+4}{(x^2+4)^2}$

crit. #s: $x^2 = 4 \to x = \pm 2$
(denom. never 0)

~~crit #~~ in $[0,4]$: $x = 2$

$f(0) = 0$, $\boxed{f(2) = \frac{2}{4+4} = \frac{1}{4}}$, $f(4) = \frac{4}{16+4} = \frac{1}{5}$

maximum

2. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

   (A) 9 (B) $\sqrt{27}$ $\boxed{\text{(C) } \sqrt{3}}$

   (D) 3 (E) No such value of c exists.

1) f cont. on $[0,3]$ ✓
2) f diff'ble on $(0,3)$ ✓
so MVT applies ✓

$f'(c) = \dfrac{f(3) - f(0)}{3-0}$

$3c^2 = \dfrac{3^3 - 0^3}{3-0} \to 3c^2 = \dfrac{27}{3} = 9 \to c^2 = 3 \to c = \pm\sqrt{3} \to c = \sqrt{3}$
($c = -\sqrt{3}$ not in $(0,3)$)

3. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

   $\boxed{\text{(A) } 1}$ (B) -2 (C) $-2,\ 1$

   (D) $-2,\ \frac{1}{2}$ (E) $-2,\ \frac{1}{2},\ 1$

$f'(x) = 6x^2 + 6x - 12$
$= 6(x^2 + x - 2)$
$= 6(x-1)(x+2) = 0$ (~~DNE~~)
$x = 1, -2$ crit. #s

2nd deriv. test:
$f''(x) = 12x + 6$: $f''(1) = 18 > 0$ → loc. min at $x = 1$
$f''(-2) = -18 < 0 \to$ ~~x~~ loc. max

4. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

   (A) 0 (B) 1 $\boxed{\text{(C) } 2}$

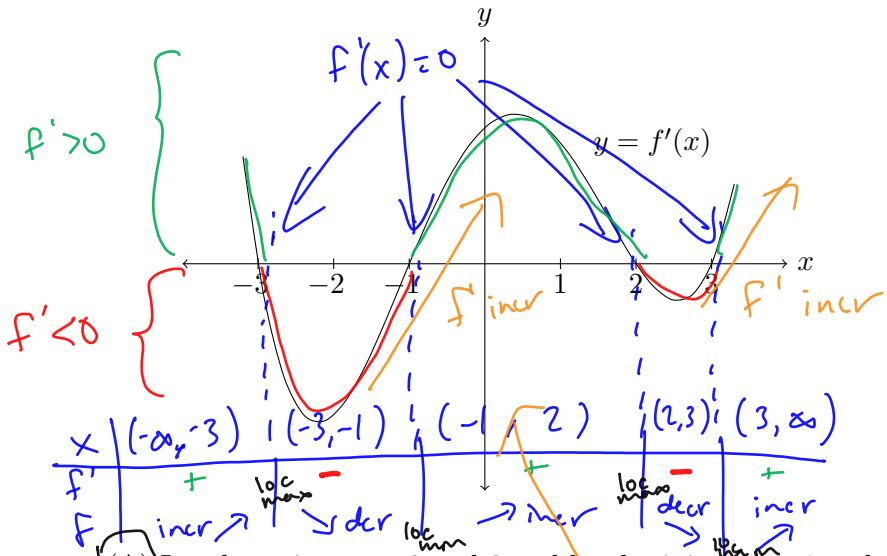
   (D) 3 (E) 4

$f'(x) = 4x^3 - 16x$
$f''(x) = 12x^2 - 16 = 0$ (or ~~DNE~~)
$x^2 = \frac{16}{12} = \frac{4}{3}$
$x = \pm\frac{2}{\sqrt{3}}$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	−	+
f	∪ c.u.	∩ c.d	∪ c.u.

inflection points (2)

5. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



x	$(-\infty,-3)$	$(-3,-1)$	$(-1,2)$	$(2,3)$	$(3,\infty)$
f'	+	−	+	−	+
f	incr ↗	loc max ↘ decr	loc min → incr	loc max ↘ decr	loc min ↗ incr

(A) Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

6. Referring to the same graph of the derivative in question 5, at approximately what x-value(s) is $f(x)$ concave up? $\Leftrightarrow f'' > 0 \Leftrightarrow f'$ increasing (2 intervals):

$(-2.1, 0.8)$ and $(2.6, \infty)$

(A) $x < -1$ and $x > 1.5$

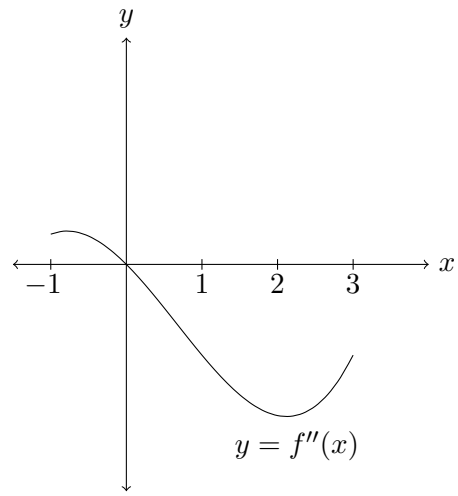
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

7. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. $f''>0$ here ✓

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. $f''<0$ here ✓

(C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to − here ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to − (f' changes ↗ to ↘) here ✓

(E) All of the above.

8. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$ (0 ~~DNE~~)

$x = 2, 1$

x	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
f''	+	−	+
f	CU	CD	CU

9. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}.$$ "$= \frac{0}{0}$" → use L'Hospital's:

(A) $+\infty$ (B) $-\infty$ (C) 0
(D) 1/2 (E) $-1/2$

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cos x}{2x}$$
$$= \text{"}\frac{1}{0}\text{"} : \frac{+}{+} \to +\infty$$

10. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}.$$ "$= \frac{0}{0}$": use L'Hospital's:

(A) 0 (B) 1 (C) $+\infty$
(D) -1 (E) 1/2

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{L'H}{=} \lim_{x\to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0$$

11. Determine the number of inflection points of the graph of $y = x^2 - \frac{1}{x}$ on its domain.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

$y' = 2x + \frac{1}{x^2} \to y'' = 2 - \frac{2}{x^3}$

$y'' = 0$ or DNE: $x = 0$ or $2 = \frac{2}{x^3} \to x^3 = 1 \to x = 1$

x	$(-\infty, 0)$	0	$(0,1)$	$(1,\infty)$
y''	+	asymp.	−	+
y	∪		∩	∪

Inflection point at $x=1$ (not at $x=0$, a vertical asymptote)

12. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 (C) 20, 40
(D) 26, 27 (E) None of the above

maximize $A = xy$ with $y + 2x = 80$: $y = 80 - 2x$, so
$A(x) = x(80-2x) = 80x - 2x^2 \to$ maximize on $(0, 40)$

$A'(x) = 80 - 4x = 0$ (or DNE)
$\to x = 20 \to y = 80 - 2x = 80 - 40$
$= 40$

2nd deriv. test:
$A''(x) = -4 < 0$ so maximum ✓

13. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) -3 (B) -2 $\boxed{\text{(C) } -1}$

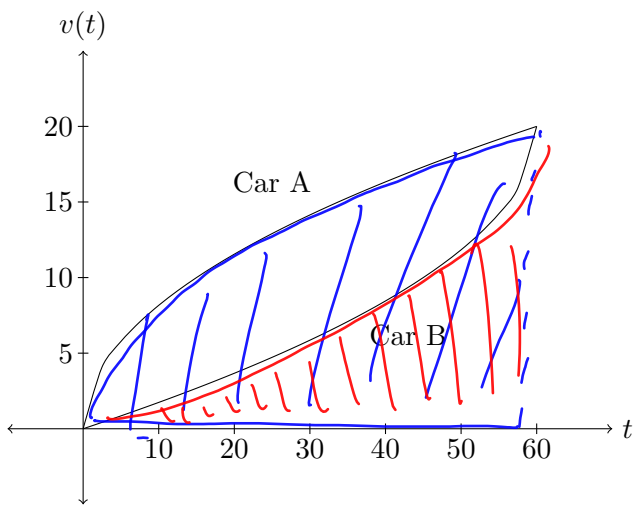
(D) 1 (E) 3

$f'(x) = 2x - \frac{3}{2}x^2 + C$, $f'(0) = C = -1 \rightarrow f'(x) = 2x - \frac{3}{2}x^2 - 1$

So $f(x) = x^2 - \frac{1}{2}x^3 - x + D$, $f(0) = D = 1 \rightarrow f(x) = x^2 - \frac{1}{2}x^3 - x + 1$

$\rightarrow f(2) = 4 - 4 - 2 + 1 = \underline{\underline{-1}}$

14. Below is the graph of the velocity (measured in ft/sec) over the interval $0 \le t \le 60$ for two cars, Car A and Car B. How do the distances traveled by each compare at over this interval?



$\boxed{\text{(A)}}$ Car A has traveled further than Car B (blue area bigger than red)

(B) Car B has traveled further than Car A

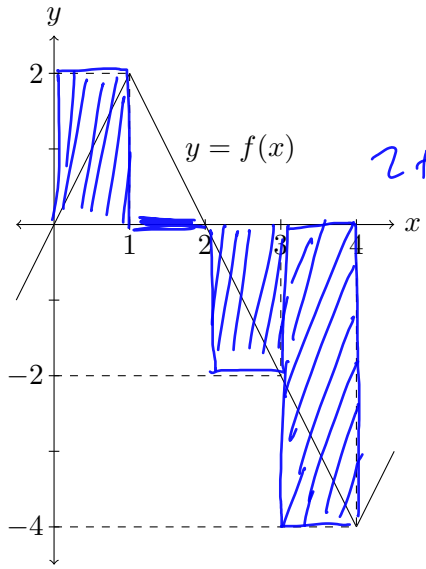
(C) Car A and Car B have traveled the same distance

(D) Cannot be determined because we don't know the equations of the cars' position curves

(E) Cannot be determined because we don't know the equations of the cars' velocity curves

15. If we use a right endpoint approximation with four subintervals (i.e., R_4), then what is the resulting approximation for

$$\int_0^4 f(x)\,dx?$$



$2+0-2-4=-4$

(A) 2 (B) -4 (C) -2

(D) 0 (E) -1

16. Evaluate the definite integral $\int_{-1}^{1}(x^2+2x+1)\,dx$. $= \left[\frac{x^3}{3}+x^2+x\right]_{-1}^{1}$

(A) 8/3 (B) -1 (C) 5/3

(D) $-5/3$ (E) 0

$= \left(\frac{1}{3}+1+1\right)-\left(-\frac{1}{3}+1-1\right)$

$= \frac{2}{3}+2 = \frac{8}{3}$

17. Assume that $\int_{-2}^{3} f(x)\,dx = 4$. What is the value of $\int_{-2}^{3}(f(x)+1)\,dx$?

(A) 4 (B) 5 (C) 6

(D) 9 (E) 20

$= \int_{-2}^{3} f(x)dx + \int_{-2}^{3} 1\,dx$

$= 4 + 1(3-(-2))$

$= 4+5 = 9$

18. Which of the following is the derivative of the function

$$f(x) = \int_1^{x^2} \frac{1}{t^3+1}\,dt?$$

(A) $\frac{2x}{x^6+1}$ (B) $\frac{1}{x^6+1}$ (C) $\frac{2x}{x^5+1}$

(D) $\frac{1}{x^3+1}$ (E) $\frac{2x}{x^3+1}$

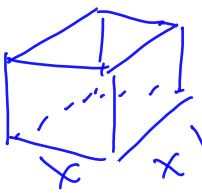
$f(x) = g(u)$ with $u(x) = x^2$ and $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$.

So $f'(x) = g'(u)\frac{du}{dx}$

$= \frac{1}{u^3+1} \cdot 2x$

$= \frac{2x}{(x^2)^3+1} = \frac{2x}{x^6+1}$

19. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?



(A) \$500 (B) \$600 (C) \$1000

(D) \$1200 (E) \$2000

$V = x^2 y$

$V = 4000$

$\to y = \frac{4000}{x^2}$

cost $= (\$1)$(area of bottom + sides)

$= x^2 + 4xy \to C(x) = x^2 + 4x\left(\frac{4000}{x^2}\right) = x^2 + \frac{16000}{x}$

minimize over $(0, \infty)$.

$C'(x) = 2x - \frac{16000}{x^2} = 0$ or DNE $\to \cancel{x = 0}$, $x = 20$

$C''(20) > 0$, so minimum ✓

$\to C(20) = 1200$

20. Find $f(x)$ if $f'(x) = 3x^2 + \frac{2}{x}$ for $x > 0$ and $f(1) = 3$.

(A) $x^3 + 2\ln x$ (B) $x^3 - \frac{1}{x} + 3$ (C) $x^3 + 2\ln x + 1$

(D) $6x + 2\ln x - 3$ (E) $x^3 + 2\ln x + 2$

$f(x) = x^3 + 2\ln x + C$

$f(1) = 1 + 2\ln 1 + C = 1 + C = 3 \to C = 2$, so $f(x) = x^3 + 2\ln x + 2$

21. Which of the following choices for the function $f(x)$ would yield a situation in which the limit could be evaluated using l'Hopital's Rule?

$\lim_{x\to\infty} \sin x$ DNE

$\lim_{x\to\infty} e^{-x} = 0$

$$\lim_{x\to\infty} \frac{f(x)}{x^2} = \text{"}\frac{?}{\infty}\text{"}$$

L'H applies if $\lim_{x\to\infty} f(x) = \infty$

(A) $\sin(x)$ (B) e^{-x} (C) $\cos(x)$

(D) $\ln(x)$ (E) All of the above

$\lim_{x\to\infty} \ln x = \infty$ ✓

$\lim_{x\to\infty} \cos x$ DNE

cont on $[0,2]$, so use closed interval method

22. A particle moves along a line with velocity $v(t) = t - \ln(t^2+1)$. What is its maximum velocity on the interval $0 \le t \le 2$?

$v'(t) = 1 - \frac{2t}{t^2+1} = 0$, DNE $\to t^2+1 = 2t$
$\to t^2 - 2t + 1 = 0$
$(t-1)^2 = 0$
$t = 1$
crit # in $[0,2]$

(A) $1 - \ln 2$ (B) 0 (C) $2 - \ln 5$

(D) $\ln 2 - 1$ (E) $\ln 5 - 2$

$v(0) = 0 - \ln(1) = 0$ $v(2) = 2 - \ln(5) \approx 0.39$
$v(1) = 1 - \ln(2) \approx 0.31$ maximum

23. If $f(1) = 9$ and $f'(x) \ge 3$ for all x in the interval $[1, 4]$, then what is the smallest possible value of $f(4)$?

using MVT: $f'(c) = \frac{f(4) - f(1)}{4-1}$ for some c in $[1,4]$, so
$3f'(c) = f(4) - f(1)$
$\to f(4) = 3f'(c) + f(1)$
$= 3f'(c) + 9$
$\ge 3(3) + 9 = 18$

(A) 19 (B) 18 (C) 12

(D) Cannot be determined (E) None of the above

24. Using the table below, identify all critical numbers for the twice differentiable function $f(x)$ and determine if each critical value is a local maximum, local minimum, or cannot be determined (CBD).

x	-7	-3	-2	0	1	4	6
$f(x)$	0	0	3	-10	0	25	2
$f'(x)$	-4	0	0	0	9	0	2
$f''(x)$	5	1	0	8	-7	-3	0
		min $x=-3$	CBD $x=-2$	min $x=0$		max $x=4$	

crit #s: $x = -3, -2, 0, 4$ (where $f'(x) = 0$)

2nd deriv test:
$f'' > 0 \to$ loc min
$f'' < 0 \to$ loc max
$f'' = 0 \to$ CBD

(A) Local max at 1 and 4; local min at -7, -3, and 0; CBD at -2 and 6

(B) Local max at -3 and 0; local min at 4; CBD at -2

(C) Local max at 4; local min at -3 and 0; CBD at -2

(D) Local max at 4; local min at 0

(E) Local max at -7, -3, and 0; local min at 1 and 4; CBD at -2 and 6