University of Connecticut
Department of Mathematics

MATH 1060Q   PRACTICE FOR EXAM 2   SPRING 2020

NAME: ______________________________

Instructor Name: ______________________ Section: ________

Read This First!

- This set of questions is provided to help you practice for exam 2. It is meant to give you an idea of the type of questions and topics covered on the exam. You should NOT expect the questions on the actual exam to be exactly the same or small variations on these.
- You will be provided with numerical solutions to each problem, as well as a reference to examples in the book to look at if you are stuck. You may ask about these questions during office hours.

Solution:

Answers are given in the following pages. It is recommended that you try the problems first on your own and then compare your own answers with these.

1. Sketch a graph of the parent function $f(x) = 4^x$ and the transformed function $f(x) = 4^{x-1} + 3$. Describe the transformations in words. What are the asymptotes of the parent function and the transformed graph?

> **Solution:** right 1, up 3. asymptote $y = 3$
>
> See example 2a and 5a in 3.1.

2. Evaluate the following logarithmic expressions.
   (a) $\log(100)$
   (b) $\log_4(16^3)$
   (c) $\ln(e^3)$
   (d) $e^{2\ln(x)}$

> **Solution:** a: 2, b: 6, c: 3, d: x^2.
>
> See example 3 and 9 in 3.2.

3. Use the change of base formula to rewrite $\log_3(316)$ in terms of ln.

> **Solution:** $\dfrac{\ln(316)}{\ln(3)}$
>
> See example 1 in 3.3.

4. Expand the logarithmic expression

$$\ln\left(\frac{\sqrt{2x+1}}{2x}\right).$$

> **Solution:** $\frac{1}{2}\ln(2x+1) - \ln(2x)$
>
> See example 5b in section 3.3.

5. Simplify the following expression into a single logarithm.

$$2\log_5(x) + \log_5(x^3+1) + 1$$

Solution: $\log_5(5x^2(x^3+1))$. Note: $1 = \log_5(5)$.
See example 6 3.3.

6. In each of the following, solve for x. Give an exact answer.
   (a) $7 - 2e^x = 5$

   Solution: $x = 0$
   See example 3 in 3.4.

   (b) $2(3^x) = 16$

   Solution: $x = \log_3(8)$ or $x = \dfrac{\ln(8)}{\ln(3)}$.
   See example 2b in 3.4

   (c) $2^{x^2} = 4^x$

   Solution: $x = 0, 2$
   Since $4 = 2^2$, we get $4^x = (2^2)^x = 2^{2x}$. So the expression becomes $2^{x^2} = 2^{2x}$. Then by the one-to-one property, $x^2 = 2x$. So $x^2 - 2x = 0$ so $x(x-2) = 0$ which means $x = 0$ or $x = 2$.

   (d) $\log_6(3x+1) - \log_6(3) = \log_6(2x)$.

   Solution: $x = 1/3$
   See example 8 and 9 in 3.4.

7. A laptop that costs \$1000 new is worth \$600 after 8 months. If its value is modeled by $y = ae^{-kt}$ where t is in months, find a and k, then find the value ter 18 months.

Solution: $a = 1000$, $k = \dfrac{-\ln(3/5)}{9}$. After 18 months, value is

$$1000e^{\frac{\ln(3/5)}{8}18}$$

See example 2 in 3.5.

8. Initially, there are 500 bacteria. After 3 hours, there are 1400. Assuming their growth is modeled by $y = ae^{kt}$, find an expression for the number of bacteria after t hours. How many are there after 9 hours?

Solution: After t hours there are

$$5000e^{\frac{\ln(14/5)}{3}t}$$

bacteria. So after 9 hours there are

$$5000e^{3\ln(14/5)} = 5000e^{\ln((14/5)^3)} = 5000(14/5)^3$$

See example 2 in 3.5.

9. Convert the angle measures below from degrees to radians or radians to degrees.
   (a) $45°$
   (b) $\pi/18$ radians
   (c) 3 radians

Solution: $\pi/4, 10°, \frac{540}{\pi}° \approx 171.89°$

See example 3 and 4 in 4.1.

10. Find several angles which are coterminal to $\pi/5$.

Solution: $11\pi/5, 21\pi/5, 31\pi/5, -9\pi/5, -19\pi/5$

See example 1 in 4.1.

11. Let $\theta =$ (any standard angle on the unit circle, or coterminal to any angle on the unit circle). Find $\sin, \cos, \sec, \csc, \tan, \cot$ of θ.

Solution: See the values on the unit circle. Be sure to know which gives you cos and which gives you sin. Then know the definition of the other 4 trig functions in terms of sin and cos.

See example 1 in 4.2.

12. If $\cos(t) = 1/5$, what is $\cos(-t + 2\pi)$? Explain your reasoning.

Solution: $\cos(-t+2\pi) = \cos(-t) = \cos(t) = 1/5$.

See example 2 in 4.2.

13. Evaluate each of the following:
    (a) $\cos(\frac{\pi}{3})$
    (b) $\tan(\frac{5\pi}{4})$

Solution: 1/2, 1

See example 1 in 4.2.

14. If $\sin(\theta) = 1/5$ and θ is in quadrant II, find the other 5 trig functions of θ.

Solution:

$$\cos(\theta) = -\sqrt{24}/5, \tan(\theta) = -1/\sqrt{24}, \cot(\theta) = -\sqrt{24}$$
$$\sec(\theta) = -5/\sqrt{24}, \csc(\theta) = 5.$$

See example 6 in 4.4.

15. If the angle of elevation of a hill is 10° and you cover 150ft of distance on the trail that goes up the hill (the hypotenuse), how much is your change in elevation?

Solution: $150\sin(10^\circ)$

See example 10 in 4.3.

16. Show that $(1+\cos(\theta))(1-\cos(\theta)) = \sin^2(\theta)$

Solution:

$$\begin{aligned} (1+\cos(\theta))(1-\cos(\theta)) &= 1-\cos(\theta)+\cos(\theta)-\cos^2(\theta) && \text{expanding} \\ &= 1-\cos^2(\theta) && \text{cancelling middle terms} \\ &= \sin^2(\theta) \end{aligned}$$

Note, in the last step, we used $\sin^2(\theta)+\cos^2(\theta) = 1$ so $1-\cos^2(\theta) = \sin^2(\theta)$.

See example 7 in 4.3.