1. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

   (A) 3 (B) 0 (C) 4

   (D) 2 $\boxed{\text{(E) 5}}$

$$f'(x) = (3x^2 - 2x)(x^4 - x + 2) + (x^3 - x^2 + 1)(4x^3 - 1)$$
$$\to f'(1) = (3-2)(1-1+2) + (1-1+1)(4-1) = (1)(2) + (1)(3) = 5$$

2. Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

   (A) $y = \frac{1}{2}$ (B) $y = -\frac{1}{2}x + 1$ (C) $y = \frac{1}{2}x$

   (D) $y = -\frac{1}{4}x + \frac{3}{4}$ $\boxed{\text{(E) } y = \frac{1}{4}x + \frac{1}{4}}$

$y' = \dfrac{1(x+1) - x(1)}{(x+1)^2}$ at $x = 1$: $y' = \dfrac{2-1}{2^2} = \dfrac{1}{4} \to$ slope $= \frac{1}{4}$

point: $y(1) = \frac{1}{1+1} = \frac{1}{2}$, so $(1, \frac{1}{2}) \longrightarrow y - \frac{1}{2} = \frac{1}{4}(x-1)$

$\to y = \frac{1}{4}x - \frac{1}{4} + \frac{1}{2}$

$\to y = \frac{1}{4}x + \frac{1}{4}$

3. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

   (A) 1 $\boxed{\text{(B) } -1}$ (C) 0

   (D) 1/2 (E) $\sqrt{2}/2$

$f^{(4)}(x) = f^{(8)}(x) = \cdots = f^{(124)}(x) = \sin x$, so

$f^{(125)}(x) = \frac{d}{dx} \sin x = \cos x$. $f^{(125)}(\pi) = \cos\pi = -1$

4. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $g(x)$?

   (A) x (B) x^2 $\boxed{\text{(C) } \sin x}$

   (D) $\sin^2 x$ (E) None of the above

$\sin^2 x = (\sin x)^2$, so $g(x) = \sin x$, $f(x) = x^2$

5. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\frac{dy}{dx}$ when $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$

at $x=2$: $(4)(1) + (3)(3)$

$= 4+9 = \boxed{13}$

(A) 9 (B) 12 $\boxed{\text{(C) } 13}$

(D) 15 (E) 23

6. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

(A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ $\boxed{\text{(B) } x_{n+1} = (x_n^2 + 7)/(2x_n)}$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

(D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$. $f(x) = x^2 - 7$, $f'(x) = 2x$, so

$X_{n+1} = X_n - \frac{x_n^2 - 7}{2x_n} = \frac{2x_n^2 - (x_n^2 - 7)}{2x_n} = \frac{x_n^2 + 7}{2x_n}$

7. If $g(x) = \frac{ax+b}{cx+d}$, then $g'(1)$ is which of the following? Note: The numbers a, b, c, and d are constants.

(A) $\frac{a+b-c-d}{c+d}$ $\boxed{\text{(B) } \frac{ad-bc}{(c+d)^2}}$ (C) $\frac{a+b-c-d}{(c+d)^2}$

(D) $\frac{ad+bc}{c+d}$ (E) $\frac{ad+bc}{(c+d)^2}$

$g'(x) = \frac{a(cx+d) - (ax+b)\cdot c}{(cx+d)^2} \Rightarrow g'(1) = \frac{a(c+d) - (a+b)c}{(c+d)^2} = \frac{ac + ad - ac - bc}{(c+d)^2}$

$= \frac{ad - bc}{(c+d)^2}$

8. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\frac{3\pi}{4}$ $\boxed{\text{(B) } \frac{3\pi}{4} + \frac{1}{2}}$ (C) $\frac{1}{2}$

(D) $\frac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

$f'(x) = 3x^2 \arctan(x) + x^3 \cdot \frac{1}{1+x^2}$

$\rightarrow f'(1) = 3(1)^2 \arctan(1) + (1)^3 \cdot \frac{1}{1+(1)^2} = 3(\frac{\pi}{4}) + \frac{1}{2}$

9. Consider the functions $f(x) = \sin(x^2)$ and $g(x) = \sin^2(x)$. Which of the following is true?

(A) $f'(x) = \cos(x^2)$ (B) $g'(x) = -2\sin(x)\cos(x)$ (C) $f'(x) = g'(x)$

(D) $f'(\pi) = g'(\pi) = 0$ $\boxed{\text{(E) } f'(0) = g'(0)}$

$f'(x) = \cos(x^2)\cdot 2x$

$f'(\pi) = \cos(\pi^2)\cdot 2\pi \neq 0$

$f'(0) = \cos(0)\cdot 2(0) = 0$

$g'(x) = 2\sin x \cos x$

$g'(\pi) = 2\sin\pi\cos\pi = 0$

$g'(0) = 2\sin 0 \cos 0 = 0$

10. If $\frac{d}{dx}[f(4x)] = x^2$, then find $f'(x)$.

$\boxed{\text{(A) } \frac{x^2}{64}}$ (B) $\frac{x^2}{16}$ (C) $\frac{x^2}{4}$

(D) x^2 (E) $4x^2$

$\frac{d}{dx}[f(4x)] = f'(4x)\cdot 4 = x^2 \rightarrow f'(4x) = \frac{x^2}{4}$

Let $u = 4x$.

Then $\frac{u}{4} = x$, so $f'(u) = \frac{\left(\frac{u}{4}\right)^2}{4} = \frac{\frac{u^2}{16}}{4} = \frac{u^2}{64}$

$\rightarrow f'(x) = \frac{x^2}{64}$

11. Find $\frac{d}{dx}[\log_4(3x)]$.

(A) $\frac{1}{3x\ln 4}$ $\boxed{\text{(B) } \frac{1}{x\ln 4}}$ (C) $\frac{1}{x}$

(D) $\frac{3}{x\ln 4}$ (E) $\frac{3}{x}$

$\frac{d}{dx}[\log_4(3x)] = \frac{1}{3x\ln 4}\cdot\frac{d}{dx}(3x) = \frac{3}{3x\ln 4} = \frac{1}{x\ln 4}$

12. Find an equation of the tangent line to the curve $(x^2+y^2)^2 = 4x^2y$ at the point $(1,1)$.

$\boxed{\text{(A) } y = 1}$ (B) $y = x$ (C) $y = 2x - 1$

(D) $y = -x + 2$ (E) $y = -2x + 3$

$\frac{d}{dx}[(x^2+y^2)^2] = \frac{d}{dx}[4x^2y]$

$2(x^2+y^2)(2x+2y\frac{dy}{dx}) = 8xy + 4x^2\frac{dy}{dx}$

at $x=1, y=1$: $2(1+1)(2+2\frac{dy}{dx}) = 8 + 4\frac{dy}{dx}$

$8 + 8\frac{dy}{dx} = 8 + 4\frac{dy}{dx}$

$4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0 \rightarrow$ slope $= 0$

$y = 0x + 1$

13. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

✓ I. $k > 0$

I. $P(5) > P(0) \to P$ increasing, so $k > 0$ ✓

✗ II. $P'(5) < 0$

II. $P'(t) = kP > 0$ since $k > 0$ and $P > 0$ so $P'(5) > 0$ ✗

✓ III. $P'(10) = 100ke^{10k}$

III. $P'(10) = ?$ $P'(t) = kP(t) = k(100e^{kt})$ $\to P'(10) = k(100e^{k(10)}) = 100ke^{10k}$ ✓

$\boxed{\text{(A) I and III only.}}$ (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

14. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

$y = Ae^{kt}$, $A = 10$.

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

At $t = 20$, $y = 5$:

$\boxed{\text{(D) } 10e^{-t\ln(2)/20}}$ (E) $10e^{t\ln(2)/20}$

$5 = 10e^{k(20)}$

$\frac{1}{2} = e^{20k} \to 20k = \ln\frac{1}{2} \to k = \frac{\ln\frac{1}{2}}{20} = \frac{-\ln 2}{20} \to \underline{y = 10e^{-\frac{\ln 2}{20}t}}$

15. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by,

$y = Ae^{kh}$ $A = 1013$

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

at $h = 1000$, $y = 0.88A$:

(D) $1000e^{-h\ln(2)/20}$ $\boxed{\text{(E) } 1013e^{h\ln(0.88)/1000}}$

$0.88\cancel{A} = \cancel{A}e^{k(1000)} \to \ln(0.88) = 1000k \to k = \frac{\ln(0.88)}{1000} \to \underline{y = 1013e^{\frac{\ln(0.88)}{1000}h}}$

16. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

$y^3 = x^4 + 11$

$\boxed{\text{(A) 27 cm/s}}$ (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

$\to 3y^2\frac{dy}{dt} = 4x^3\frac{dx}{dt}$

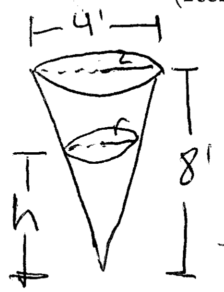
When $x = 2, y = 3, \frac{dy}{dt} = 32$:

$3(3)^2(32) = 4(2)^3\frac{dx}{dt} \to \frac{dx}{dt} = \frac{27 \cdot 32}{4 \cdot 8} = \underline{27 \text{ cm/s}}$

$\frac{dV}{dt} = -2$

17. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3}r^2h$?)

$\frac{dh}{dt} = ?$ $h = 2$



(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

$\boxed{\text{(D) } \frac{8}{\pi} \text{ ft/min}}$ (E) $\frac{16}{\pi}$ ft/min

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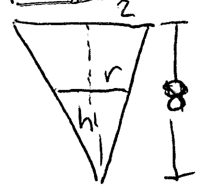

$\frac{r}{h} = \frac{2}{8} \to r = \frac{h}{4}$

$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h$

$V = \frac{\pi}{48}h^3$

$\frac{dV}{dt} = \frac{\pi}{16}h^2\frac{dh}{dt}$

$-2 = \frac{\pi}{16}(2)^2\frac{dh}{dt}$

$\to -\frac{16}{2\pi} = \frac{dh}{dt} = -\frac{8}{\pi}$

18. Determine $f''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

$f'(x) = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4}$

(A) $\frac{-1}{2x^2}$ (B) $\frac{6\ln x}{x^4}$ (C) $\frac{1-6\ln x}{x^4}$

(D) $\frac{1-2\ln x}{x^3}$ $\boxed{\text{(E) None of the above}}$

$f'(x) = \frac{x - 2x\ln x}{x^4}$

$\to f''(x) = \frac{[1-(2\ln x + 2x\cdot\frac{1}{x})]x^4 - (x-2x\ln x)\cdot 4x^3}{x^{16}} = \frac{x^4[1-2\ln x + 2 - 4 + 8\ln x]}{x^{16}} = \frac{-1+6\ln x}{x^{12}}$

19. Use the linearization for the function $f(x) = \sqrt{x^3+2x+1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ $\boxed{\text{(C) } \frac{17}{8}}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

$L(x) = f'(1)(x-1) + f(1)$

$f'(x) = \frac{1}{2}(x^3+2x+1)^{-1/2}(3x^2+2)$

$f'(1) = \frac{1}{2}\left(\frac{1}{\sqrt{4}}\right)(5) = \frac{5}{4}$

$f(1) = \sqrt{1+2+1} = \sqrt{4} = 2$

$\to L(x) = \frac{5}{4}(x-1)+2.$ $f(1.1) \approx L(1.1) = \frac{5}{4}(0.1)+2$

$= \frac{5}{40} + 2 = \frac{1}{8} + \frac{16}{8}$

$= \frac{17}{8}$

20. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 $\boxed{\text{(B) } 19/6}$ (C) 15/4

(D) 12/7 (E) 17/6

$f'(x) = 2x \to f'(3) = 6$

$f(3) = 9 - 10 = -1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \left(\frac{-1}{6}\right) = 3 + \frac{1}{6} = \frac{19}{6}$