University of Connecticut
Department of Mathematics

Math 1131 Practice Problems for Exam 2

Sections Covered: 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 3.9, 3.10, 4.8

Read This First!

- Please arrive early and bring a pencil and eraser.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.
- **Calculators are NOT allowed** on the exam. No books or other references or electronic devices are permitted.

1. Determine $f'(1)$ for the function $f(x) = (x^3 - x^2 + 1)(x^4 - x + 2)$.

   (A) 3 (B) 0 (C) 4

   (D) 2 (E) 5

2. Find the equation of the tangent line to the curve $y = \dfrac{x}{x+1}$ at $x = 1$.

   (A) $y = \dfrac{1}{2}$ (B) $y = -\dfrac{1}{2}x + 1$ (C) $y = \dfrac{1}{2}x$

   (D) $y = -\dfrac{1}{4}x + \dfrac{3}{4}$ (E) $y = \dfrac{1}{4}x + \dfrac{1}{4}$

3. If $f(x) = \sin(x)$, determine $f^{(125)}(\pi)$.

   (A) 1 (B) -1 (C) 0

   (D) $1/2$ (E) $\sqrt{2}/2$

4. To compute the derivative of $\sin^2 x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $g(x)$?

   (A) x (B) x^2 (C) $\sin x$

   (D) $\sin^2 x$ (E) None of the above

5. Let $y = f(x)g(x)$. Using the table of values below, determine the value of $\dfrac{dy}{dx}$ when $x = 2$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	2	4	4
2	3	4	1	3
3	2	3	2	2
4	4	1	5	5
5	1	5	3	1

(A) 9 (B) 12 (C) 13

(D) 15 (E) 23

6. What is the recursion from Newton's method for solving $x^2 - 7 = 0$?

(A) $x_{n+1} = (x_n^3 - 9x_n)/(x_n^2 - 7)$ (B) $x_{n+1} = (x_n^2 + 7)/(2x_n)$ (C) $x_{n+1} = (x_n^2 - 7)/(2x_n)$

(D) $x_{n+1} = (3x_n^2 + 7)/(2x_n)$ (E) $x_{n+1} = (3x_n^2 - 7)/(2x_n)$

7. If $g(x) = \dfrac{ax + b}{cx + d}$, then $g'(1)$ is which of the following? Note: The numbers a, b, c, and d are constants.

(A) $\dfrac{a + b - c - d}{c + d}$ (B) $\dfrac{ad - bc}{(c + d)^2}$ (C) $\dfrac{a + b - c - d}{(c + d)^2}$

(D) $\dfrac{ad + bc}{c + d}$ (E) $\dfrac{ad + bc}{(c + d)^2}$

8. For the function $f(x) = x^3 \arctan(x)$, which of the following is $f'(1)$?

(A) $\dfrac{3\pi}{4}$ (B) $\dfrac{3\pi}{4} + \dfrac{1}{2}$ (C) $\dfrac{1}{2}$

(D) $\dfrac{\pi}{4}$ (E) $3\tan(1) + \sec^2(1)$

9. Consider the functions $f(x) = \sin(x^2)$ and $g(x) = \sin^2(x)$. Which of the following is true?

(A) $f'(x) = \cos(x^2)$ (B) $g'(x) = -2\sin(x)\cos(x)$ (C) $f'(x) = g'(x)$

(D) $f'(\pi) = g'(\pi) = 0$ (E) $f'(0) = g'(0)$

10. If $\dfrac{d}{dx}\left[f(4x)\right] = x^2$, then find $f'(x)$.

(A) $\dfrac{x^2}{64}$ (B) $\dfrac{x^2}{16}$ (C) $\dfrac{x^2}{4}$

(D) x^2 (E) $4x^2$

11. Find $\dfrac{d}{dx}\left[\log_4(3x)\right]$.

(A) $\dfrac{1}{3x\ln 4}$ (B) $\dfrac{1}{x\ln 4}$ (C) $\dfrac{1}{x}$

(D) $\dfrac{3}{x\ln 4}$ (E) $\dfrac{3}{x}$

12. Find an equation of the tangent line to the curve $(x^2+y^2)^2 = 4x^2y$ at the point $(1,1)$.

(A) $y = 1$ (B) $y = x$ (C) $y = 2x - 1$

(D) $y = -x + 2$ (E) $y = -2x + 3$

13. The size of a colony of bacteria at time t hours is given by $P(t) = 100e^{kt}$, where P is measured in millions. If $P(5) > P(0)$, then determine which of the following is true.

I. $k > 0$

II. $P'(5) < 0$

III. $P'(10) = 100ke^{10k}$

(A) I and III only. (B) I and II only. (C) I only.

(D) II only. (E) I, II, and III.

14. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time t is given by

(A) $10e^{10k}$ (B) $\ln(10)e^{kt/10}$ (C) $\ln(10)e^{t/10}$

(D) $10e^{-t\ln(2)/20}$ (E) $10e^{t\ln(2)/20}$

15. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by 12% for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height h is given by,

(A) $1000e^{10h}$ (B) $\ln(1013)e^{kh/12}$ (C) $1013e^{\ln(0.88)/1000}$

(D) $1000e^{-h\ln(2)/20}$ (E) $1013e^{h\ln(0.88)/1000}$

16. A particle moves along the curve $y = \sqrt[3]{x^4 + 11}$. As it reaches the point $(2, 3)$, the y-coordinate is increasing at a rate of 32 cm/s. Which of the following represents the rate of increase of the x-coordinate at that instant?

(A) 27 cm/s (B) 9 cm/s (C) 27/2 cm/s

(D) 67/4 cm/s (E) None of the above

17. Water is withdrawn at a constant rate of 2 ft^3/min from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft, and the height of the tank is 8 ft. How fast is the water level falling when the depth of the water in the tank is 2 ft? (Remember that the volume of a cone of height h and radius r is $V = \frac{\pi}{3}r^2h$?)

(A) $\frac{2}{\pi}$ ft/min (B) $\frac{4}{\pi}$ ft/min (C) $\frac{6}{\pi}$ ft/min

(D) $\frac{8}{\pi}$ ft/min (E) $\frac{16}{\pi}$ ft/min

18. Determine $f''(x)$ for the function $f(x) = \frac{\ln x}{x^2}$.

(A) $\frac{-1}{2x^2}$ (B) $\frac{6\ln x}{x^4}$ (C) $\frac{1-6\ln x}{x^4}$

(D) $\frac{1-2\ln x}{x^3}$ (E) None of the above

19. Use the linearization for the function $f(x) = \sqrt{x^3+2x+1}$ at $x = 1$ to approximate the value of $f(1.1)$.

(A) $\frac{161}{80}$ (B) $\frac{21}{10}$ (C) $\frac{17}{8}$

(D) $\frac{1}{2}$ (E) $\frac{17}{16}$

20. Let $f(x) = x^2 - 10$. If $x_1 = 3$ in Newton's method to solve $f(x) = 0$, determine x_2.

(A) 1/2 (B) 19/6 (C) 15/4

(D) 12/7 (E) 17/6