1. The distance traveled by a particle in t seconds is given by $s(t) = t^2 + 3t$. What is the particle's average velocity over the interval $1 \le t \le 4$? [1]

   $\boxed{\text{(A) } 8}$ (B) 0 (C) 2

   (D) 5 (E) -1

$$\frac{s(4) - s(1)}{4 - 1} = \frac{(4^2+3\cdot 4) - (1^2+3\cdot 1)}{3}$$

$$= \frac{16+12-1-3}{3}$$

$$= \frac{24}{3} = \underline{\underline{8}}$$

2. Evaluate the following limit: [1]

$$\lim_{x \to 1^-} \frac{x-3}{x-1}.$$

   (A) 2 (B) -2 (C) -1

   (D) $+\infty$ (E) $-\infty$

$x = 0.999$: $\frac{x-3}{x-1} = \frac{-2.001}{-0.001} = 2001$

large, positive

$\to \underline{\underline{\infty}}$

3. Using the table below, what appears to be the value of the limit [1]

$$\lim_{x \to 2^+} f(x)$$

x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
$f(x)$	3	7	291	4081	?	-9532	-112	-17	-1

$-\infty \leftarrow$

(A) ∞ $\boxed{\text{(B) } -\infty}$ (C) 0

looks like $\lim_{x \to 2^+} f(x) = -\infty$

(D) -1000 (E) None of the above.

4. If $\lim_{x \to 3^+} f(x) = 5$ what can be said about $\lim_{x \to 3^-} f(x)$? [1]

(A) It must be 5 (B) It must be $f(3)$ (C) It must be $f(5)$

(D) It must be -5 $\boxed{\text{(E) It cannot be determined}}$

two one-sided limits match if $\lim_{x \to 3} f(x)$ exists, but we don't know if it does here.

5. If $-x^2 - x + 1 \leq g(x) \leq x^2 - x + 1$ for all $x \neq 0$, what is $\lim_{x \to 0} g(x)$? [1]

(A) 0 $\boxed{\text{(B) } 1}$ (C) 2

(D) $g(0)$ (E) Cannot be determined

$$\lim_{x \to 0} (-x^2 - x + 1) \leq \lim_{x \to 0} g(x) \leq \lim_{x \to 0} (x^2 - x + 1)$$

$$\rightarrow \quad 1 \leq \lim_{x \to 0} g(x) \leq 1 \rightarrow \lim_{x \to 0} g(x) = 1$$

6. Evaluate the following limit: [1]

$$\lim_{x \to 4} \frac{x^2 - 8x + 16}{x - 4}.$$

$\boxed{\text{(A) } 0}$ (B) 8 (C) -8

(D) $+\infty$ (E) $-\infty$

$$\lim_{x \to 4} \frac{x^2 - 8x + 16}{x - 4} = \lim_{x \to 4} \frac{(x-4)^2}{x-4} = \lim_{x \to 4} (x-4) = 4 - 4 = \underline{\underline{0}}$$

7. If $\lim_{x \to 1} f(x) = 3$, $\lim_{x \to 1} g(x) = -2$, and $\lim_{x \to 1} h(x) = 4$, evaluate the limit [1]

$$\lim_{x \to 1} \left(\frac{2f(x)}{g(x)} + \sqrt{h(x)} \right).$$

$\boxed{\text{(A) } -1}$ (B) 3 (C) 13

(D) 5 (E) 7

$$\lim_{x \to 1} \left(\frac{2f(x)}{g(x)} + \sqrt{h(x)} \right)$$

$$= \frac{2 \lim_{x \to 1} f(x)}{\boxed{\lim_{x \to 1} g(x)}} + \sqrt{\lim_{x \to 1} h(x)} = \frac{2 \cdot 3}{-2} + \sqrt{4}$$

(not 0)

$$= -3 + 2 = \underline{\underline{-1}}$$

8. If the function $f(x)$ is continuous on the interval $[-1, 3]$, $f(-1) = 1$, and $f(3) = 11$, which numbers below are guaranteed to be values of $f(x)$ by the Intermediate Value Theorem on the interval $(-1, 3)$? [1]

I. 3

II. $\sqrt{2}$

III. 3π

(A) I only (B) II only (C) III only

(D) I and II only $\boxed{\text{(E) I, II, and III}}$

all values between $f(-1)=1$ and $f(3)=11$ are guaranteed. $1<\sqrt{2}<3<3\pi<11$,

so all 3 are guaranteed

9. Determine the value of the number k that makes the function $f(x)$ below continuous: [1]

$$f(x) = \begin{cases} 1 - kx & \text{if } x < 1, \\ k + x & \text{if } x \geq 1. \end{cases}$$

$\boxed{\text{(A) } 0}$ (B) 1 (C) $-3/4$

(D) $1/2$ (E) $15/17$

want $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$:

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} (1-kx) = 1-k \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} (k+x) = k+1 \end{aligned}\right\} \rightarrow 1-k = k+1$$

$0 = 2k$

$\underline{k = 0}$

10. Consider the function [1]

$$h(x) = \begin{cases} \frac{1}{x} & \text{if } 0 < x < 1, \\ x & \text{if } x > 1. \end{cases}$$

Which of the following are true?

✓ I. $\lim_{x\to 1^+} h(x)$ exists

✓ II. $\lim_{x\to 1^-} h(x)$ exists

✓ III. $\lim_{x\to 1} h(x)$ exists

~~IV.~~ $h(x)$ is continuous at $x = 1$

(A) I only (B) I and II only $\boxed{\text{(C) I, II, and III only}}$

(D) IV only (E) I, II, III, and IV

I. $\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} \frac{1}{x} = \frac{1}{1} = 1$ ✓

II. $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} x = 1$ ✓

III. $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^+} h(x) = 1$, so $\lim_{x\to 1} h(x) = 1$ ✓

IV. $h(1)$ not defined, so h not continuous at $x=1$

11. Evaluate the following limit: [1]

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{x}.$$

(A) $+\infty$ (B) $-\infty$ (C) 0

$\boxed{\text{(D) } 1}$ (E) -1

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{\cancel{x}} = \lim_{x\to\infty} \frac{\sqrt{x^2+2}}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{x^2+2}}{1} \cdot \frac{1}{x}$$

$$= \lim_{x\to\infty} \sqrt{x^2+2} \cdot \frac{1}{\sqrt{x^2}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{x^2+2}{x^2}} = \lim_{x\to\infty} \sqrt{1 + \frac{2}{x^2}}$$

12. The function $f(x) = \dfrac{x^2+1}{x^3+8}$ has which of the following? [1]

(A) no vertical or horizontal asymptotes

(B) 1 vertical asymptote and 1 horizontal asymptote

(C) 2 vertical asymptotes and 1 horizontal asymptote

(D) 1 vertical asymptote and 2 horizontal asymptotes

(E) 1 vertical asymptote and no horizontal asymptotes

• Vertical: $x^3+8=0 \to x^3=-8 \to x=-2$ (and $x^2+1=5 \neq 0$ at $x=-2$, so Vert. asymptote there)

$\to$ 1 vertical

• Horizontal: $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2+1}{x^3+8} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim\limits_{x\to\pm\infty} \dfrac{\frac{1}{x}+\frac{1}{x^3}}{1+\frac{8}{x^3}} = \dfrac{0+0}{1+0} = 0 \to$ horiz. asym. $y=0$

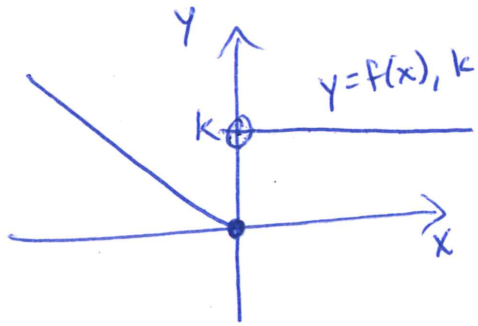
$\to$ 1 horizontal

13. For what value of the number k is the following function differentiable at $x=0$? [1]

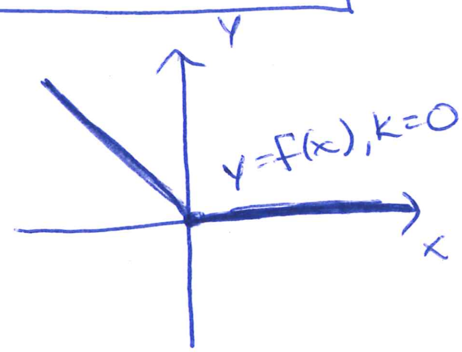
$$f(x) = \begin{cases} -x & x \leq 0 \\ k & x > 0 \end{cases}$$

(A) -2 (B) -1 (C) 0

(D) 1 (E) No value of k makes this function differentiable at $x=0$



not continuous at x=0, so not differentiable



not locally linear at x=0, so not ~~continuous~~ differentiable

14. If $f(x) = 3x^{10}$, then $\lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$ is which of the following? [1]

(A) $f'(x)$ $\boxed{\text{(B) } f'(1)}$ (C) Does not exist

(D) 0 (E) None of the above

$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \underline{f'(1)}$ by limit def'n of derivative

$\left(f'(x) = 3 \cdot 10\, x^9 = 30x^9, \text{ so } \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = f'(1) = 30 \cdot 1^9 = \underline{\underline{30}} \right)$

15. If we want to calculate the derivative $f'(x)$ of $f(x) = 3x + 4$ using the limit definition of the derivative which of the following limits do we need to evaluate and to what does the limit evaluate? [1]

$\boxed{\text{(A) } \lim_{h\to 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 3}$

(B) $\lim_{h\to 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 0$

(C) $\lim_{h\to 0} \frac{3h + 4 - (3x+4)}{h} = 3x + 3$

(D) $\lim_{h\to 0} \frac{3(x+h) + 4 - (3h+4)}{h} = 3$

(E) None of the above.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)+4-(3x+4)}{h}$$

$$= \lim_{h\to 0} \frac{3x+3h+4-3x-4}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = \underline{\underline{3}}$$

16. Below is the graph of the derivative $g'(x)$ of a function $g(x)$. [1]

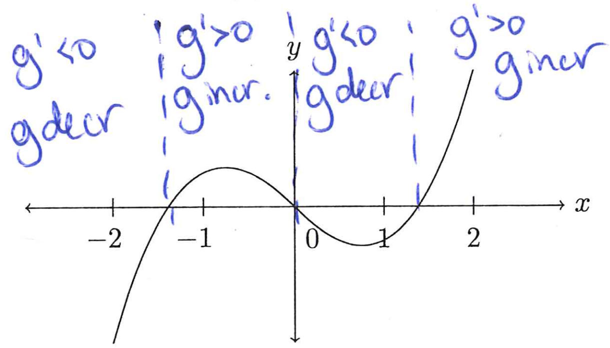
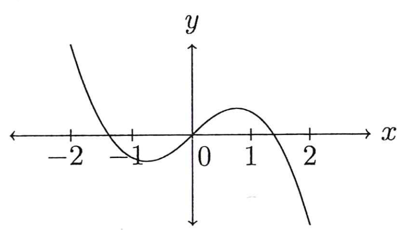


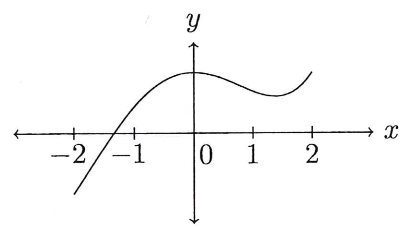
Figure 1: Graph of $g'(x)$.

Which of the following is a possible graph of $g(x)$?

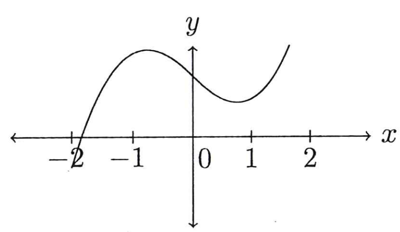
(A)



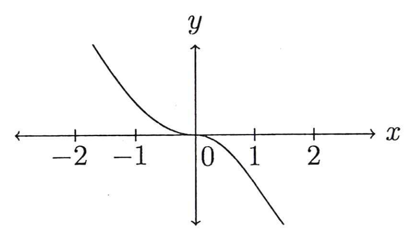
(B)



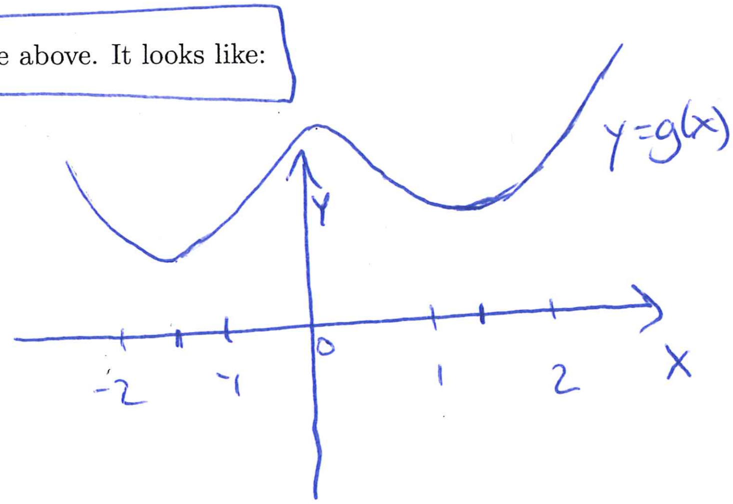
(C)



(D)



$\boxed{\text{(E) None of the above. It looks like:}}$



17. If $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ for $x > 0$, then $f'(4)$ is which of the following? [1]

(A) $\dfrac{5}{4}$ (D) $\dfrac{3}{4}$ $\boxed{\text{(C) } \dfrac{3}{16}}$

(B) $\dfrac{255}{32}$ (E) $\dfrac{257}{32}$

$$f(x) = x^{1/2} + x^{-1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} + \left(-\frac{1}{2}\right)x^{-3/2}$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}\right)$$

$$= \frac{1}{2} \cdot \frac{x-1}{x\sqrt{x}}$$

$$f'(4) = \frac{1}{2} \cdot \frac{4-1}{4\sqrt{4}} = \underline{\underline{\frac{3}{16}}}$$