§14.5 The Chain Rule Practice Exercises

1. Find dz/dt if $z = x^2 + y^2 + xy$, $x = \sin t$, and $y = e^t$.

2. Find dw/dt if $w = xe^{y/z}$, $x = t^2$, $y = 1 - t$, and $z = 1 + 2t$.

3. If $z = f(x, y)$ and f is differentiable with $x = g(t)$ and $y = h(t)$, use the following table of values to compute dz/dt at $t = 3$.

$$\begin{array}{lll} g(3) = 2 & g'(3) = 5 & f_x(2, 7) = 6 \\ h(3) = 7 & h'(3) = -4 & f_y(2, 7) = -8 \end{array}$$

4. Find $\partial w/\partial r$ and $\partial w/\partial \theta$ at $r = 2$ and $\theta = \pi/2$ if $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, and $z = r\theta$.

Solutions

1. $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt} = (2x+y)\cos t + (2y+x)e^t = (2\sin t + e^t)\cos t + (2e^t + \sin t)e^t$

2. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\cdot\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\cdot\dfrac{dz}{dt} = e^{y/z}(2t) + \dfrac{xe^{y/z}}{z}(-1) + xe^{y/z}\left(-\dfrac{y}{z^2}\right)(2)$

$$= 2te^{(1-t)/(1+2t)} - \frac{t^2 e^{(1-t)/(1+2t)}}{1+2t} - \frac{2t^2(1-t)e^{(1-t)/(1+2t)}}{(1+2t)^2}$$

3. At $t=3$, $x=2$ and $y=7$.

$$\left.\frac{dz}{dt}\right|_{t=3} = \left.\frac{\partial z}{\partial x}\right|_{x=2,y=7}\cdot\left.\frac{dx}{dt}\right|_{t=3} + \left.\frac{\partial z}{\partial y}\right|_{x=2,y=7}\cdot\left.\frac{dy}{dt}\right|_{t=3}$$
$$= f_x(2,7)g'(3) + f_y(2,7)h'(3)$$
$$= 6(5) + (-8)(-4) = 62$$

4. $\dfrac{\partial w}{\partial r} = \dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial r} + \dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial r} + \dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial r}$

$= (y+z)\cos\theta + (x+z)\sin\theta + (x+y)\theta$

$= (r\sin\theta + r\theta)\cos\theta + (r\cos\theta + r\theta)\sin\theta + (r\cos\theta + r\sin\theta)\theta$

$= (2+\pi)(0) + (0+\pi)(1) + (0+2)(\pi/2) = 2\pi$

$\dfrac{\partial w}{\partial \theta} = \dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial \theta} + \dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial \theta} + \dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial \theta}$

$= (y+z)(-r\sin\theta) + (x+z)r\cos\theta + (x+y)r$

$= (r\sin\theta + r\theta)(-r\sin\theta) + (r\cos\theta + r\theta)r\cos\theta + (r\cos\theta + r\sin\theta)r$

$= (2+\pi)(-2) + (0+\pi)(0) + (0+2)(2) = -2\pi.$