University of Connecticut
Department of Mathematics

MATH 1131 EXAM 3 PRACTICE PROBLEMS FALL 2019

Sections Covered: 4.1-4.4, 4.7, 4.9, 5.1-5.3

Read This First!

- The exam will take place in your discussion section meeting on **Thursday, November 14.** Please arrive early and bring a pencil and eraser.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.
- **Calculators are NOT allowed** on the exam. No books or other references or electronic devices are permitted.

1. Which of the following is the absolute maximum value of the function $f(x) = \dfrac{x}{x^2+4}$ on the interval $[0, 4]$?

(A) $\frac{1}{8}$ (B) $\frac{1}{5}$ $\boxed{\text{(C) } \frac{1}{4}}$

(D) $\frac{1}{2}$ (E) 1

$f'(x) = \dfrac{(x^2+4)1 - x(2x)}{(x^2+4)^2} = \dfrac{-x^2+4}{(x^2+4)^2}$

crit #s: $x^2 = 4 \to x = \pm 2$

(denom. never 0)

crit # in $[0,4]$: $x = 2$

$f(0) = 0$, $\boxed{f(2) = \frac{2}{4+4} = \frac{1}{4}}$, $f(4) = \frac{4}{16+4} = \frac{1}{5}$

maximum

2. Find all value(s) of the number c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[0, 3]$, if any exist.

(A) 9 (B) $\sqrt{27}$ $\boxed{\text{(C) } \sqrt{3}}$

(D) 3 (E) No such value of c exists.

1) f cont. on $[0,3]$ ✓

2) f diff'ble on $(0,3)$ ✓

so MVT applies:

$f'(c) = \dfrac{f(3) - f(0)}{3 - 0}$

$3c^2 = \dfrac{3^3 - 0^3}{3 - 0} \to 3c^2 = \dfrac{27}{3} = 9 \to c^2 = 3 \to c = \pm\sqrt{3} \to c = \sqrt{3}$

($c = -\sqrt{3}$ not in $(0,3)$)

3. Find all value(s) of x where $f(x) = 2x^3 + 3x^2 - 12x$ has a local minimum.

$\boxed{\text{(A) } 1}$ (B) -2 (C) $-2, 1$

(D) $-2, \frac{1}{2}$ (E) $-2, \frac{1}{2}, 1$

$f'(x) = 6x^2 + 6x - 12$

$= 6(x^2 + x - 2)$

$= 6(x-1)(x+2) = 0$ (or ~~DNE~~)

$x = 1, -2$ crit #s

2nd derivative test:

$f''(x) = 12x + 6 \to f''(1) = 18 > 0$ ✓ local min at $x=1$

$f''(-2) = -18 < 0$ ✗ local max

4. How many inflection points does the graph of $f(x) = x^4 - 8x^2 - 7$ have?

(A) 0 (B) 1 $\boxed{\text{(C) } 2}$

(D) 3 (E) 4

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16 = 0$ (or ~~DNE~~)

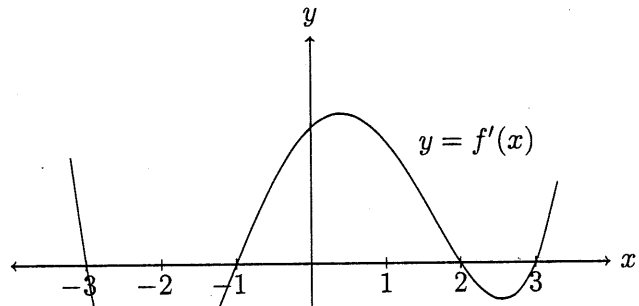
$x^2 = \frac{16}{12} = \frac{4}{3}$

$x = \pm\frac{2}{\sqrt{3}}$

x	$(-\infty, -\frac{2}{\sqrt{3}})$	$(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$	$(\frac{2}{\sqrt{3}}, \infty)$
f''	+	−	+
f	c.u.	c.d.	c.u.

inflection points at $x = \pm\frac{2}{\sqrt{3}}$ (2)

5. Below is the graph of the *derivative* $f'(x)$ of a function $f(x)$. At what x-value(s) does $f(x)$ have a local maximum or local minimum?



x	$(-\infty,-3)$	$(-3,-1)$	$(-1,2)$	$(2,3)$	$(3,\infty)$
f'	+	−	+	−	+
f	↗	loc max ↘	loc min ↗	loc max ↘	loc min ↗

(A) Local maxima at -3 and 2 and local minima at -1 and 3

(B) Local maxima at -1 and 3 and local minima at -3 and 2

(C) Local maxima at -1 and 3 and local minimum at 2

(D) Local maxima at -3 and 2 and local minimum at -1

(E) None of the above

x	$(-\infty,-2.1)$	$(-2.1,0.8)$	$(0.8,2.6)$	$(2.6,\infty)$
f''	−	+	−	+
f'	decr	incr	decr	incr
f	conc. down	C.U.	C.D	C.U.

6. Referring to the same graph of the derivative in question 5, at approximately what x-value(s) is $f(x)$ concave up?

(A) $x < -1$ and $x > 1.5$

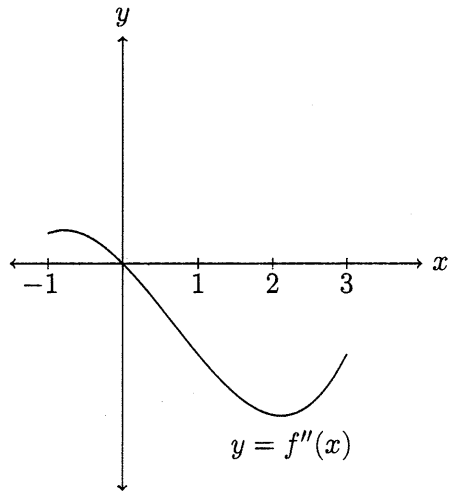
(B) $-1 < x < 2$

(C) $-2.1 < x < .8$ and $x > 2.6$

(D) $-\infty < x < \infty$

(E) We cannot determine concavity of $f(x)$ from the graph of $f'(x)$.

7. Below is the graph of the *second derivative* $f''(x)$ of a function $f(x)$ on the interval $[-1, 3]$. Which of the following statements must be true?



(A) The function $f(x)$ is concave up when $-1 < x < 0$. $f''>0$ here ✓

(B) The derivative $f'(x)$ is decreasing when $0 < x < 3$. $f''<0$ here ✓

(C) The function $f(x)$ has a point of inflection at $x = 0$. f'' changes + to − here ✓

(D) The derivative $f'(x)$ has a local maximum at $x = 0$. derivative of f' changes + to − (f' changes ↗ to ↘) here ✓

(E) All of the above.

8. On which interval(s) is the function $f(x) = x^4 - 6x^3 + 12x^2 + 1$ concave down?

(A) $(-\infty, 1)$ only (B) $(1, 2)$ only (C) $(-\infty, -1)$ and $(2, \infty)$

(D) $(2, \infty)$ only (E) $(-\infty, 1)$ and $(2, \infty)$

(or DNE)

$f'(x) = 4x^3 - 18x^2 + 24x$

$f''(x) = 12x^2 - 36x + 24 = 12(x^2 - 3x + 2) = 12(x-2)(x-1) = 0$

$x = 2, 1$

x	$(-\infty,1)$	$(1,2)$	$(2,\infty)$
f''	+	−	+
f	∪	∩ C.D. on $(1,2)$	∪

9. Evaluate the following limit:

$$\lim_{x\to 0^+} \frac{\sin x}{x^2}.$$

"$\frac{0}{0}$": use L'Hôpital's:

$\boxed{\text{(A) } +\infty}$ (B) $-\infty$ (C) 0

(D) 1/2 (E) $-1/2$

$$\lim_{x\to 0^+} \frac{\sin x}{x^2} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\cos x}{2x} = \text{"}\frac{1}{0}\text{"} : \frac{+}{+} \to +\infty$$

10. Evaluate the following limit:

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}.$$

"$\frac{0}{0}$": use L'Hôpital's:

$\boxed{\text{(A) } 0}$ (B) 1 (C) $+\infty$

(D) -1 (E) 1/2

$$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{L'H}{=} \lim_{x\to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \boxed{0}$$

11. Determine the number of inflection points of the graph of $y = x^2 - \frac{1}{x}$ on its domain.

(A) 0 $\boxed{\text{(B) } 1}$ (C) 2 (D) 3 (E) 4

$y' = 2x + \frac{1}{x^2} \to y'' = 2 - \frac{2}{x^3}$

$y''=0$, DNE: $x=0$ or $2 = \frac{2}{x^3} \to x^3 = 1 \to x = 1$

x	$(-\infty,0)$	asymptote	$(0,1)$	$(1,\infty)$
y''	+		−	+
y	∪		∩	∪

inflection point at $x=1$ (not at $x=0 \to$ asymptote)

12. Find two positive numbers x and y satisfying $y + 2x = 80$ whose product is a maximum.

(A) 24, 32 (B) 26, 28 $\boxed{\text{(C) } 20, 40}$

(D) 26, 27 (E) None of the above

maximize $A = xy$ with $y + 2x = 80$: $y = 80 - 2x$, so $A(x) = x(80-2x) = 80x - 2x^2$.

maximize on $(0,\infty)$

$A'(x) = 80 - 4x = 0$ (or ~~DNE~~)

$\to x = 20 \to y = 80 - 2x = 80 - 40 = 40 = y$

2nd deriv. test:

$A''(x) = -4 < 0$ so ∩ maximum! ✓

13. A certain function $f(x)$ satisfies $f''(x) = 2 - 3x$ with $f'(0) = -1$ and $f(0) = 1$. Compute $f(2)$.

(A) -3 (B) -2 $\boxed{\text{(C) } -1}$

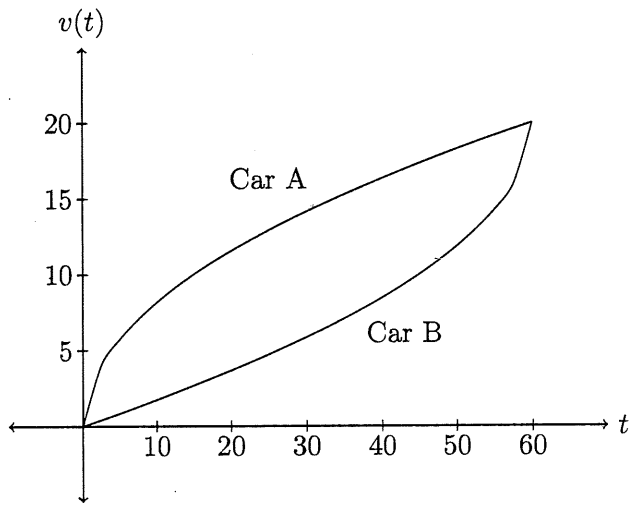
(D) 1 (E) 3

$f'(x) = 2x - \frac{3}{2}x^2 + C$, $f'(0) = C = -1 \rightarrow f'(x) = 2x - \frac{3}{2}x^2 - 1$

so $f(x) = x^2 - \frac{1}{2}x^3 - x + D$, $f(0) = D = 1 \rightarrow f(x) = x^2 - \frac{1}{2}x^3 - x + 1$

$f(2) = 4 - 4 - 2 + 1 = -1$

14. Below is the graph of the velocity (measured in ft/sec) over the interval $0 \leq t \leq 60$ for two cars, Car A and Car B. How do the distances traveled by each compare at over this interval?



$\boxed{\text{(A)}}$ Car A has traveled further than Car B (area under "car A" curve is bigger)

(B) Car B has traveled further than Car A

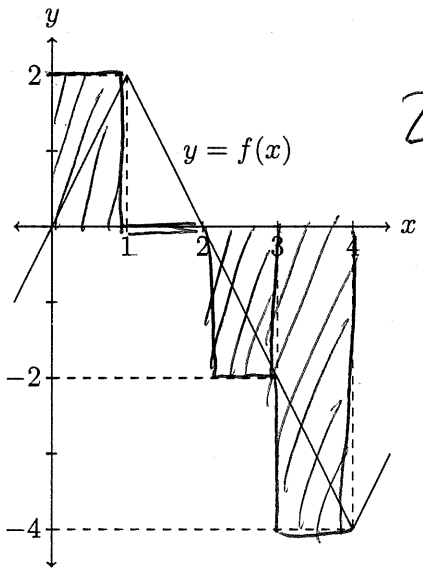
(C) Car A and Car B have traveled the same distance

(D) Cannot be determined because we don't know the equations of the cars' position curves

(E) Cannot be determined because we don't know the equations of the cars' velocity curves

15. If we use a right endpoint approximation with four subintervals (i.e., R_4), then what is the resulting approximation for

$$\int_0^4 f(x)\,dx?$$



$2+0-2-4=-4$

(A) 2 (B) −4 (C) −2

(D) 0 (E) −1

16. Evaluate the definite integral $\int_{-1}^{1}(x^2+2x+1)\,dx$. $= \left[\frac{x^3}{3}+x^2+x\right]_{-1}^{1}$

(A) 8/3 (B) −1 (C) 5/3

(D) −5/3 (E) 0

$= \left(\frac{1}{3}+1+1\right)-\left(-\frac{1}{3}+1-1\right)$

$= \frac{2}{3}+2 = \frac{8}{3}$

17. Assume that $\int_{-2}^{3} f(x)\,dx = 4$. What is the value of $\int_{-2}^{3}(f(x)+1)\,dx$?

(A) 4 (B) 5 (C) 6

(D) 9 (E) 20

$\int_{-2}^{3}(f(x)+1)\,dx = \int_{-2}^{3} f(x)\,dx + \int_{-2}^{3} 1\,dx$

$= 4 + (3-(-2))\cdot 1$

$= 4+5 = \underline{9}$

18. Which of the following is the derivative of the function

$$f(x) = \int_1^{x^2} \frac{1}{t^3+1}\,dt?$$

$\boxed{\text{(A) } \frac{2x}{x^6+1}}$ (B) $\frac{1}{x^6+1}$ (C) $\frac{2x}{x^5+1}$

(D) $\frac{1}{x^3+1}$ (E) $\frac{2x}{x^3+1}$

$f(x) = g(u)$ with $u(x) = x^2$ and $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$.

so $f'(x) = g'(u)\frac{du}{dx}$

$= \frac{1}{u^3+1} \cdot 2x$

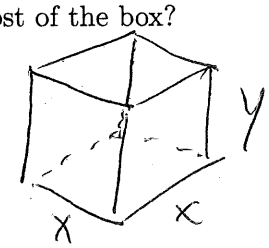
$= \frac{1}{(x^2)^3+1} \cdot 2x$

$= \frac{2x}{x^6+1}$

19. A box with square base and open top must have a volume of 4000 cm^3. If the cost of the material used is \$1/cm^2, then what is the smallest possible cost of the box?

(A) \$500 (B) \$600 (C) \$1000

$\boxed{\text{(D) \$1200}}$ (E) \$2000



$V = x^2 y = 4000$

$\to y = \frac{4000}{x^2}$

cost = (\$1)(area of bottom + sides)

$= x^2 + 4xy \to C(x) = x^2 + 4x\left(\frac{4000}{x^2}\right) = x^2 + \frac{16000}{x}$

$\to$ minimize over domain $(0, \infty)$:

$C'(x) = 2x - \frac{16000}{x^2} = 0$ or DNE: $\cancel{x=0}$ or $2x^3 = 16000 \to x^3 = 8000 \to \underline{x = 20}$

$C''(x) = 2 + \frac{32000}{x^3} > 0$ at $x = 20$, so minimum ✓

$\to C(20) = 400 + \frac{16000}{20}$

$= 400 + 800$

$= \underline{1200}$

is minimum cost

20. Find $f(x)$ if $f'(x) = 3x^2 + \frac{2}{x}$ for $x > 0$ and $f(1) = 3$.

(A) $x^3 + 2\ln x$ (B) $x^3 - \frac{1}{x} + 3$ (C) $x^3 + 2\ln x + 1$

(D) $6x + 2\ln x - 3$ $\boxed{\text{(E) } x^3 + 2\ln x + 2}$

$f(x) = x^3 + 2\ln x + C$

$f(1) = 1 + 2\ln 1 + C = 1 + C = 3$

$\to C = 2$

so $\underline{f(x) = x^3 + 2\ln x + 2}$