Math 1131 Week 7 Worksheet

Name: __________________________

Discussion Section:______________

Solutions should show all of your work, not just a single final answer.

3.9: Related Rates

1. The radius r of a spherical balloon is expanding at the constant rate of 14 in/min.

   (a) Determine the rate at which the volume V changes with respect to time, in in^3/min, when $r = 8$ in. Round your answer to the nearest integer. Recall $V = \frac{4}{3}\pi r^3$.

   (b) Determine the rate at which the surface area S changes with respect to time, in in^2/min, when $r = 8$ in. Round your answer to the nearest integer. Recall $S = 4\pi r^2$.

   (c) If the radius doubles, does dV/dt double? Does dS/dt double?

2. Water is flowing into an upside-down right circular cone with height 3 m and radius 2 m at the top. As water fills the cone, let the height of the water in the cone be h m, and let r m be the radius of the top of the water.

   (a) Draw and label a diagram of this scenario, find an expression for r in terms of h, then find an expression for the volume V of water in the cone in terms of h alone (no r in the formula). Recall the volume of a right-circular cone with height h and radius r is $\frac{1}{3}\pi r^2 h$.

   (b) Express dV/dt in terms of h and dh/dt. If dV/dt is constant (and not zero), explain from your formula why dh/dt cannot be constant as well.

   (c) Assuming the water flows into the cone at a constant rate of 2 m^3/min, how quickly is its height changing, in m/min, when the height is 2 m? Round your answer to the nearest tenth.

3. A cop sits in a parked car 10 feet from a straight road. As you drive along the road, the cop aims a radar gun at your car. Let s be the distance from your car to the cop in feet. The radar gun measures the rate at which your distance from the cop is changing with respect to time, which is ds/dt. Let x be the distance, in feet, of your car from the point on the road that is closest to the cop, so your car's velocity is dx/dt.

   (a) Draw and label a diagram of this scenario, and write $\frac{dx}{dt}$ in terms of $\frac{ds}{dt}$, s, and x.

   (b) Use part a to explain why $\frac{ds}{dt} < \frac{dx}{dt}$. Thus the radar gun's measurement of $\frac{ds}{dt}$ always *underestimates* your car's velocity. (This is why if the radar gun measures a speed greater than the speed limit on the road, the driver deserves a ticket.)

   (c) Does the conclusion in part (b) depend on the cop's car being 10 feet, rather than some other (positive) distance, from the road?

3.10: Linear Approximations and Differentials

4. (a) Find the linearization of the function $f(x) = \sqrt{x}$ at 9.

   (b) Use the linear approximation obtained in part (a) (no other methods) to approximate $\sqrt{9.2}$. Your answer based on that linearization can be given either as an exact fraction or rounded to four digits after the decimal point.

5. (a) Find the linearization of the function $f(x) = \dfrac{1}{1+x^2}$ at 7.

   (b) Use the linear approximation obtained in part (a) (no other methods) to approximate $\dfrac{1}{37}$. Round your answer to three digits after the decimal point.

6. The side length of a cube is measured to be $x = 1.3$ feet, with an error of at most 1 inch.

   (a) (No calculus) Determine the difference, in ft^3, between the volume of the cube computed with the measured side length and the volume computed with the largest (resp., smallest) value for the side length in the error range. Remember first to convert all lengths to feet! Round your final answers to three digits after the decimal point.

   (b) Use differentials to estimate the error in calculating the volume of the cube using the measured value and error estimate for the side length of the cube. Round your final answer to three digits after the decimal point.

4.8: Newton's Method

7. Apply Newton's method to estimate the solution of $x^3 - x - 1 = 0$ by taking $x_1 = 1$ and finding the least n such that x_n and x_{n+1} agree to three digits after the decimal point.

8. The number π is a solution of $\sin x = 0$ close to 3 (see Figure 1). You will use Newton's method for $\sin x = 0$ to create numerical estimates for π.

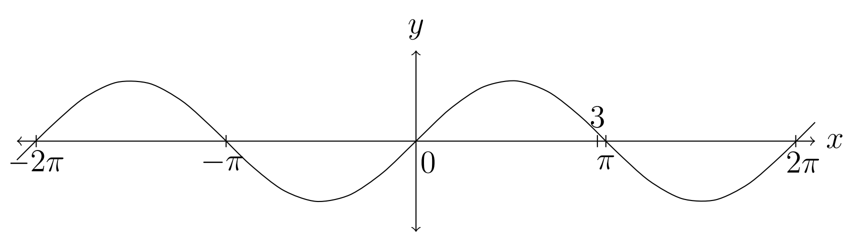

Figure 1: Graph of $y = \sin x$.

(a) Write out the recursion for Newton's method used to solve $\sin x = 0$.

(b) Using Newton's method for $\sin x = 0$ with $x_1 = 3$, find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point. (Use radians, *not* degrees!)

(c) For the n you found in part (b), to how many digits after the decimal point does x_n actually agree with π?

9. In Figure 2 is the graph of $f(x) = \ln(x) - 1$ for $0 < x < 4$. It crosses the x-axis at $x = e$. You will use Newton's method for $f(x) = 0$ to create numerical estimates for e.

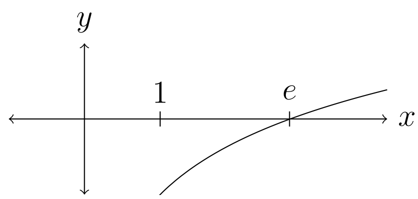

Figure 2: Graph of $y = \ln(x) - 1$.

(a) Using Newton's method for the equation $\ln(x) - 1 = 0$ with $x_1 = 1$, tabulate x_n to find the first n for which x_n and x_{n+1} agree to 5 digits after the decimal point.

(b) For the n you found in part (a), to how many digits after the decimal point does x_n actually agree with e?