

Name: \_\_\_\_\_

Discussion Section: \_\_\_\_\_

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**Solutions should show all of your work, not just a single final answer.**

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## 3.6: Derivatives of Logarithmic Functions

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1. Compute  $dy/dx$ , using logarithmic differentiation for part c. Write your final answers entirely in terms of  $x$ .

(a)  $y = \ln(2 + \sin x)$

(b)  $y = \ln(\ln x)$

(c)  $y = 5^x$

2. T/F (with justification) If  $f(x) = \ln(x^2)$  for all  $x > 0$  then  $f'(x) = \frac{1}{x^2}$ .

3. T/F (with justification) If  $f(x) = 10^x$  for all  $x$  then  $f'(x) = x10^{x-1}$ .

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## 3.8: Exponential Growth and Decay

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4. In 1859, 24 rabbits were released into the wild in Australia, where they had no natural predators. Their population grew exponentially, doubling every 6 months.
- (a) Determine  $P(t)$ , the function that gives the population at time  $t$ , **and** the differential equation describing the population growth. Let units for  $t$  be years.
- (b) After how many years, rounded to one digit after the decimal point, did the rabbit population reach 1,000,000?
- (c) Determine the *rate* of population change, in rabbits/year, midway through the third year. (**Warning:**  $t$  is not 3.5, just like the year midway through the 21st century is not 2150.) Round the final answer to 2 digits after the decimal point.

5. The element Unobtainium has a half-life of 3 years. Let  $M(t)$  be the mass of Unobtainium at time  $t$  starting with an initial amount of 14 kg.

(a) Give a formula for  $M(t)$ .

(b) After how many years will the initial mass of Unobtainium shrink to 1 kg? Round your answer to one digit after the decimal point.

6. Starbucks serves coffee at  $170^\circ$  and the room temperature in Starbucks is  $70^\circ$ . The coffee cools to  $100^\circ$  after 10 minutes. Let  $T(t)$  be the temperature of the coffee at time  $t$ , measured in minutes.

(a) Write down the differential equation for  $T(t)$  and determine a formula for  $T(t)$ .

(b) From the time when the temperature is  $100^\circ$  at  $t = 10$ , how many *additional* minutes will it take for the temperature of the coffee to reach  $80^\circ$ ? Round your answer to one digit after the decimal point.

7. T/F (with justification) If  $\frac{dy}{dx} = y$  then  $y = 0$  or  $y = e^x$ .

8. T/F (with justification) A function  $y(t)$  satisfying  $\frac{dy}{dt} = -.01y$  has  $\lim_{t \rightarrow \infty} y(t) = 0$ .