Sections Covered: 3.2, 3.3, 3.4, 3.5, 3.6, 3.8, 3.9, 3.10, 4.8

## Read This First!

- The exam will take place in your discussion section meeting on Thursday, October 17. Please arrive early and bring a pencil and eraser.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer.
- On the exam, indicate your answers on the answer sheet. The answer sheet is the ONLY place that counts as your official answers.
- Calculators are NOT allowed on the exam. No books or other references or electronic devices are permitted.

1. Determine $f^{\prime}(1)$ for the function $f(x)=\left(x^{3}-x^{2}+1\right)\left(x^{4}-x+2\right)$.
(A) 3
(B) 0
(C) 4
(D) 2
(E) 5
2. Find the equation of the tangent line to the curve $y=\frac{x}{x+1}$ at $x=1$.
(A) $y=\frac{1}{2}$
(B) $y=-\frac{1}{2} x+1$
(C) $y=\frac{1}{2} x$
(D) $y=-\frac{1}{4} x+\frac{3}{4}$
(E) $y=\frac{1}{4} x+\frac{1}{4}$
3. If $f(x)=\sin (x)$, determine $f^{(125)}(\pi)$.
(A) 1
(B) -1
(C) 0
(D) $1 / 2$
(E) $\sqrt{2} / 2$
4. To compute the derivative of $\sin ^{2} x$ with the chain rule by writing this function as a composition $f(g(x))$, what is the "inner" function $f(x)$ ?
(A) $x$
(B) $x^{2}$
(C) $\sin x$
(D) $\sin ^{2} x$
(E) None of the above
5. Let $y=f(x) g(x)$. Using the table of values below, determine the value of $\frac{d y}{d x}$ when $x=2$.

| $x$ | $f(x)$ | $f^{\prime}(x)$ | $g(x)$ | $g^{\prime}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 2 | 4 | 4 |
| 2 | 3 | 4 | 1 | 3 |
| 3 | 2 | 3 | 2 | 2 |
| 4 | 4 | 1 | 5 | 5 |
| 5 | 1 | 5 | 3 | 1 |

(A) 9
(B) 12
(C) 13
(D) 15
(E) 23
6. What is the recursion from Newton's method for solving $x^{2}-7=0$ ?
(A) $x_{n+1}=\left(x_{n}^{3}-9 x_{n}\right) /\left(x_{n}^{2}-7\right)$
(B) $x_{n+1}=\left(x_{n}^{2}+7\right) /\left(2 x_{n}\right)$
(C) $x_{n+1}=\left(x_{n}^{2}-7\right) /\left(2 x_{n}\right)$
(D) $x_{n+1}=\left(3 x_{n}^{2}+7\right) /\left(2 x_{n}\right)$
(E) $x_{n+1}=\left(3 x_{n}^{2}-7\right) /\left(2 x_{n}\right)$
7. If $g(x)=\frac{a x+b}{c x+d}$, then $g^{\prime}(1)$ is which of the following? Note: The numbers $a, b, c$, and $d$ are constants.
(A) $\frac{a+b-c-d}{c+d}$
(B) $\frac{a d-b c}{(c+d)^{2}}$
(C) $\frac{a+b-c-d}{(c+d)^{2}}$
(D) $\frac{a d+b c}{c+d}$
(E) $\frac{a d+b c}{(c+d)^{2}}$
8. For the function $f(x)=x^{3} \arctan (x)$, which of the following is $f^{\prime}(1)$ ?
(A) $\frac{3 \pi}{4}$
(B) $\frac{3 \pi}{4}+\frac{1}{2}$
(C) $\frac{1}{2}$
(D) $\frac{\pi}{4}$
(E) $3 \tan (1)+\sec ^{2}(1)$
9. Consider the functions $f(x)=\sin \left(x^{2}\right)$ and $g(x)=\sin ^{2}(x)$. Which of the following is true?
(A) $f^{\prime}(x)=\cos \left(x^{2}\right)$
(B) $g^{\prime}(x)=-2 \sin (x) \cos (x)$
(C) $f^{\prime}(x)=g^{\prime}(x)$
(D) $f^{\prime}(\pi)=g^{\prime}(\pi)=0$
(E) $f^{\prime}(0)=g^{\prime}(0)$
10. If $\frac{d}{d x}[f(4 x)]=x^{2}$, then find $f^{\prime}(x)$.
(A) $\frac{x^{2}}{64}$
(B) $\frac{x^{2}}{16}$
(C) $\frac{x^{2}}{4}$
(D) $x^{2}$
(E) $4 x^{2}$
11. Find $\frac{d}{d x}\left[x^{\ln x}\right]$.
(A) $(\ln x) x^{\ln x}$
(B) $2(\ln x) x^{(\ln x)+1}$
(C) $x^{(\ln x)-1}$
(D) $2(\ln x) x^{\ln x}$
(E) $2(\ln x) x^{(\ln x)-1}$
12. Find an equation of the tangent line to the curve $\left(x^{2}+y^{2}\right)^{2}=4 x^{2} y$ at the point $(1,1)$.
(A) $y=1$
(B) $y=x$
(C) $y=2 x-1$
(D) $y=-x+2$
(E) $y=-2 x+3$
13. The size of a colony of bacteria at time $t$ hours is given by $P(t)=100 e^{k t}$, where $P$ is measured in millions. If $P(5)>P(0)$, then determine which of the following is true.

$$
\text { I. } \quad k>0
$$

II. $P^{\prime}(5)<0$
III. $P^{\prime}(10)=100 k e^{10 k}$
(A) I and III only.
(B) I and II only.
(C) I only.
(D) II only. (E) I, II, and III.
14. Suppose that the half-life of a certain substance is 20 days and there are initially 10 grams of the substance. The amount of the substance remaining after time $t$ is given by
(A) $10 e^{10 k}$
(B) $\ln (10) e^{k t / 10}$
(C) $\ln (10) e^{t / 10}$
(D) $10 e^{-t \ln (2) / 20}$
(E) $10 e^{t \ln (2) / 20}$
15. Atmospheric pressure (the pressure of air around you) decreases as your height above sea level increases. It decreases exponentially by $12 \%$ for every 1000 meters. The pressure at sea level is 1013 hecto pascals. The amount of pressure at any height $h$ is given by,
(A) $1000 e^{10 h}$
(B) $\ln (1013) e^{k h / 12}$
(C) $1013 e^{\ln (0.88) / 1000}$
(D) $1000 e^{-h \ln (2) / 20}$
(E) $1013 e^{h \ln (0.88) / 1000}$
16. A particle moves along the curve $y=\sqrt[3]{x^{4}+11}$. As it reaches the point $(2,3)$, the $y$-coordinate is increasing at a rate of $32 \mathrm{~cm} / \mathrm{s}$. Which of the following represents the rate of increase of the $x$-coordinate at that instant?
(A) $27 \mathrm{~cm} / \mathrm{s}$
(B) $9 \mathrm{~cm} / \mathrm{s}$
(C) $13.5 \mathrm{~cm} / \mathrm{s}$
(D) $6.75 \mathrm{~cm} / \mathrm{s}$
(E) None of the above
17. Water is withdrawn at a constant rate of $2 \mathrm{ft}^{3} / \mathrm{min}$ from an inverted cone-shaped tank (meaning the vertex is at the bottom). The diameter of the top of the tank is 4 ft , and the height of the tank is 8 ft . How fast is the water level falling when the depth of the water in the tank is 2 ft ? (Remember that the volume of a cone of height $h$ and radius $r$ is $V=\frac{\pi}{3} r^{2} h$ ?)
(A) $\frac{2}{\pi} \mathrm{ft} / \mathrm{min}$
(B) $\frac{4}{\pi} \mathrm{ft} / \mathrm{min}$
(C) $\frac{6}{\pi} \mathrm{ft} / \mathrm{min}$
(D) $\frac{8}{\pi} \mathrm{ft} / \mathrm{min}$
(E) $\frac{16}{\pi} \mathrm{ft} / \mathrm{min}$
18. Determine $f^{\prime \prime}(x)$ for the function $f(x)=\frac{\ln x}{x^{2}}$.
(A) $\frac{-1}{2 x^{2}}$
(B) $\frac{6 \ln x}{x^{4}}$
(C) $\frac{1-6 \ln x}{x^{4}}$
(D) $\frac{1-2 \ln x}{x^{3}}$
(E) None of the above
19. Use the linearization for the function $f(x)=\sqrt{x^{3}+2 x+1}$ at $x=1$ to approximate the value of $f(1.1)$.
(A) 2.0125
(B) 2.10
(C) 2.125
(D) 0.5
(E) 1.925
20. Let $f(x)=x^{2}-10$. If $x_{1}=3$ in Newton's method to solve $f(x)=0$, determine $x_{2}$.
(A) $1 / 2$
(B) $19 / 6$
(C) $15 / 4$
(D) $12 / 7$
(E) $17 / 6$

