

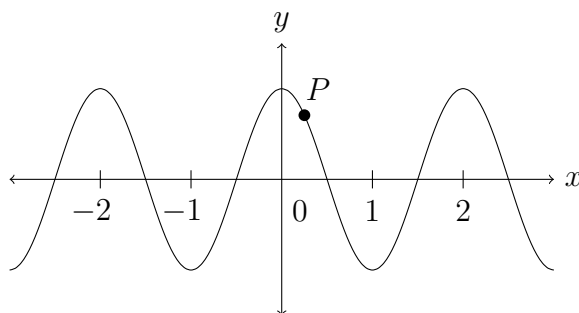
Name: _____

Discussion Section: _____

Solutions should show all of your work, not just a single final answer.

2.1: The Tangent and Velocity Problems

1. The point $P = (1/4, 1/\sqrt{2})$ lies on the curve $y = \cos(\pi x)$ where x is in **radians**, as shown below.



- (a) If $Q = (x, \cos(\pi x))$ then use your calculator to find the slope of the secant line PQ , rounded to four digits after the decimal point, for the following values of x :
- | | |
|---------------|--------------|
| (i) 0.24, | (iv) 0.26, |
| (ii) 0.249, | (v) 0.251, |
| (iii) 0.2499, | (vi) 0.2501. |
- (b) Using the results of part(a), estimate the value of the slope of the tangent line to the curve at $(1/4, 1/\sqrt{2})$ to three digits after the decimal point.

- (c) Using the estimated slope from part(b), what is an estimate for the equation of the tangent line to the graph of $y = \cos(\pi x)$ at $(1/4, 1/\sqrt{2})$? Write the final answer in the form $y = mx + b$ where m and b are each rounded to three digits after the decimal point.

2. The displacement of an object on a line, in meters, is $s = 1 + 2t + \frac{1}{4}t^2$, where t is in seconds.

- (a) Find the average velocity in m/sec over each of the following time periods. For parts (i) through (v), *round your answer to three digits after the decimal point*. In part (vi), h is a nonzero variable and the final answer is in terms of h .

(i) $[1, 1.5]$

(ii) $[1, 1.1]$

(iii) $[1, 1.01]$

(iv) $[\cdot 9, 1]$

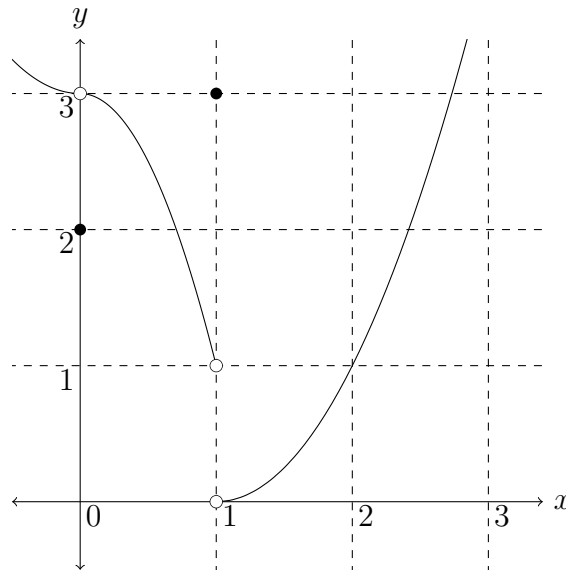
(v) $[\cdot 99, 1]$

(vi) $[1, 1 + h]$ (for $h > 0$) or $[1 + h, 1]$ (for $h < 0$)

- (b) Use the work in part a to estimate the instantaneous velocity of the object at time $t = 1$, in m/sec.

2.2: The Limit of a Function

3. The graph of $y = f(x)$ is below. Use it to compute each limit or explain why it doesn't exist.



- | | |
|-------------------------------------|-----------------------------------|
| (a) $\lim_{x \rightarrow 0^-} f(x)$ | (g) $\lim_{x \rightarrow 0} f(x)$ |
| (b) $\lim_{x \rightarrow 1^-} f(x)$ | (h) $\lim_{x \rightarrow 1} f(x)$ |
| (c) $\lim_{x \rightarrow 2^-} f(x)$ | (i) $\lim_{x \rightarrow 2} f(x)$ |
| (d) $\lim_{x \rightarrow 0^+} f(x)$ | (j) $f(0)$ |
| (e) $\lim_{x \rightarrow 1^+} f(x)$ | (k) $f(1)$ |
| (f) $\lim_{x \rightarrow 2^+} f(x)$ | (l) $f(2)$ |

4. Determine whether the following limits are finite, ∞ , or $-\infty$. If the limit does not exist for any other reason, write DNE with a justification.

(a) $\lim_{x \rightarrow 1} \frac{\sqrt{x}}{2(x-1)^2}$

(b) $\lim_{x \rightarrow 1^+} \frac{x-2}{x-1}$

(c) $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{x^2}$

5. T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph of $y = \frac{x^2 - 1}{x^2 - 2x + 1}$.

6. T/F (with justification) The line $x = 1$ is a vertical asymptote of the graph of $y = \frac{x^2 - 2x + 1}{x^2 - 1}$.

2.3: Calculating Limits Using the Limit Laws

7. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1, \\ 4 & \text{if } x = 1, \\ x + 2 & \text{if } 1 < x \leq 2, \\ 6 - x & \text{if } x > 2. \end{cases}$$

(a) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$.

(b) Evaluate the following limits if they exist. (If a limit does not exist, write DNE.)

(i) $\lim_{x \rightarrow 1^-} f(x)$

(iv) $\lim_{x \rightarrow 2^-} f(x)$

(ii) $\lim_{x \rightarrow 1^+} f(x)$

(v) $\lim_{x \rightarrow 2^+} f(x)$

(iii) $\lim_{x \rightarrow 1} f(x)$

(vi) $\lim_{x \rightarrow 2} f(x)$

8. Evaluate the following limits exactly using algebra and limit laws (some limits may be DNE).

$$(a) \lim_{x \rightarrow 2} \frac{x^3 - 2}{2x^2 - 3x + 2}$$

$$(b) \lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$$

$$(c) \lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 40} - 7}{x - 3}$$

$$(d) \lim_{x \rightarrow -2} \sqrt{x^4 + 3x + 6}$$

$$(e) \lim_{x \rightarrow 1} \frac{x^2 + 4x}{x^2 + 3x - 4}$$

$$(f) \lim_{x \rightarrow 1} \frac{(x^2 + x)^2 - 4}{x^2 + x - 2}$$

9. Evaluate the following limits using algebra and limit laws (some limits may be DNE). Note that a represents a constant, and answers may be in terms of a .

(a) $\lim_{t \rightarrow 0} \frac{\sqrt{a+t} - \sqrt{a-t}}{t}$ for $a > 0$

(b) $\lim_{h \rightarrow 0} \frac{1/(a+h)^2 - 1/a^2}{h}$ for $a \neq 0$

10. T/F (with justification) If $\lim_{x \rightarrow 2} g(x) = 0$ and $\lim_{x \rightarrow 2} h(x) = 0$ then $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$ does not exist.