



University of Connecticut
Department of Mathematics

MATH 1131

PRACTICE PROBLEMS FOR EXAM 1

FALL 2019

NAME:

Solutions

SIGNATURE: _____

Instructor Name: _____

Lecture Section: _____

TA Name: _____

Discussion Section: _____

Read This First!

- This problem set is not meant to provide complete coverage for the exam. It is designed for you to use as a diagnostic tool. If you struggle with a problem here, that gives you a topic to review and work several related problems. You should also be using homework, worksheets, and quizzes as a guide to determine what topics to review for the exam.
- The exam format and instructions will be similar to those in this document.
- Please read each question carefully. All questions are multiple choice. There is only one correct choice for each answer. Each question is one point.
- Indicate your answers on the answer sheet. The answer sheet is the **ONLY** place that counts as your official answers.
 - (1) When you're done, hand in **both** the exam booklet and the answer sheet.
 - (2) You will receive the exam booklet back after the exam is graded. The booklet is not graded, but you may circle answers there for your records.
- **No calculators are allowed.** No books or other references are permitted.

1. The distance traveled by a particle in t seconds is given by $s(t) = t^2 + 3t$. What is the particle's average velocity over the interval $1 \leq t \leq 4$? [1]

(A) 8 (B) 0 (C) 2
(D) 5 (E) -1

$$\begin{aligned}\frac{s(4) - s(1)}{4 - 1} &= \frac{(4^2 + 3 \cdot 4) - (1^2 + 3 \cdot 1)}{3} \\ &= \frac{16 + 12 - 1 - 3}{3} \\ &= \frac{24}{3} = \underline{\underline{8}}\end{aligned}$$

2. Evaluate the following limit:

$$\lim_{x \rightarrow 1^-} \frac{x - 3}{x - 1}$$

(A) 2 (B) -2 (C) -1
(D) $+\infty$ (E) $-\infty$

$$x = 0.999: \quad \frac{x - 3}{x - 1} = \frac{-2.001}{-0.001} = 2001$$

large,
positive

$\rightarrow \underline{\underline{\infty}}$

3. Using the table below, what appears to be the value of the limit

[1]

$$\lim_{x \rightarrow 2^+} f(x)$$

x	1.9	1.99	1.999	1.9999	2	2.0001	2.001	2.01	2.1
$f(x)$	3	7	291	4081	?	-9532	-112	-17	-1

- (A) ∞ (B) $-\infty$ (C) 0
 (D) -1000 (E) None of the above.

\leftarrow looks like $\lim_{x \rightarrow 2^+} f(x) = -\infty$

4. If $\lim_{x \rightarrow 3^+} f(x) = 5$ what can be said about $\lim_{x \rightarrow 3^-} f(x)$?

[1]

- (A) It must be 5 (B) It must be $f(3)$ (C) It must be $f(5)$
 (D) It must be -5 (E) It cannot be determined

two one-sided limits match if $\lim_{x \rightarrow 3} f(x)$

exists, but we don't know if it does here.

5. If $-x^2 - x + 1 \leq g(x) \leq x^2 - x + 1$ for all $x \neq 0$, what is $\lim_{x \rightarrow 0} g(x)$?

[1]

- (A) 0 (B) 1 (C) 2
 (D) $g(0)$ (E) Cannot be determined

$$\lim_{x \rightarrow 0} (-x^2 - x + 1) \leq \lim_{x \rightarrow 0} g(x) \leq \lim_{x \rightarrow 0} (x^2 - x + 1)$$

$$\rightarrow 1 \leq \lim_{x \rightarrow 0} g(x) \leq 1 \rightarrow \lim_{x \rightarrow 0} g(x) = 1$$

6. Evaluate the following limit:

[1]

$$\lim_{x \rightarrow 4} \frac{x^2 - 8x + 16}{x - 4}.$$

(A) 0

(B) 8

(C) -8

(D) $+\infty$ (E) $-\infty$

$$\lim_{x \rightarrow 4} \frac{x^2 - 8x + 16}{x - 4} = \lim_{x \rightarrow 4} \frac{(x-4)^2}{x-4} = \lim_{x \rightarrow 4} (x-4) = 4-4 = \underline{\underline{0}}$$

7. If $\lim_{x \rightarrow 1} f(x) = 3$, $\lim_{x \rightarrow 1} g(x) = -2$, and $\lim_{x \rightarrow 1} h(x) = 4$, evaluate the limit

[1]

$$\lim_{x \rightarrow 1} \left(\frac{2f(x)}{g(x)} + \sqrt{h(x)} \right).$$

(A) -1

(B) 3

(C) 13

(D) 5

(E) 7

$$\lim_{x \rightarrow 1} \left(\frac{2f(x)}{g(x)} + \sqrt{h(x)} \right)$$

$$= \frac{2 \lim_{x \rightarrow 1} f(x)}{\lim_{x \rightarrow 1} g(x)} + \sqrt{\lim_{x \rightarrow 1} h(x)} = \frac{2 \cdot 3}{-2} + \sqrt{4}$$

$$= -3 + 2 = \underline{\underline{-1}}$$

(not 0)

8. If the function $f(x)$ is continuous on the interval $[-1, 3]$, $f(-1) = 1$, and $f(3) = 11$, which numbers below are guaranteed to be values of $f(x)$ by the Intermediate Value Theorem on the interval $(-1, 3)$? [1]

I. 3

II. $\sqrt{2}$

III. 3π

(A) I only (B) II only (C) III only

(D) I and II only (E) I, II, and III

all values between $f(-1)=1$ and $f(3)=11$
are guaranteed. $1 < \sqrt{2} < 3 < \cancel{4} < 3\pi < 11$,
so all 3 are guaranteed

9. Determine the value of the number k that makes the function $f(x)$ below continuous: [1]

$$f(x) = \begin{cases} 1 - kx & \text{if } x < 1, \\ k + x & \text{if } x \geq 1. \end{cases}$$

- (A) 0 (B) 1 (C) $-3/4$
(D) $1/2$ (E) $15/17$

want $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) =$

$$\begin{aligned} \lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} (1 - kx) = 1 - k \\ \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} (k + x) = k + 1 \end{aligned} \quad \left. \vphantom{\begin{aligned} \lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^-} (1 - kx) = 1 - k \\ \lim_{x \rightarrow 1^+} f(x) &= \lim_{x \rightarrow 1^+} (k + x) = k + 1 \end{aligned}} \right\} \begin{aligned} 1 - k &= k + 1 \\ 0 &= 2k \\ k &= \underline{\underline{0}} \end{aligned}$$

10. Consider the function

[1]

$$h(x) = \begin{cases} \frac{1}{x} & \text{if } 0 < x < 1, \\ x & \text{if } x > 1. \end{cases}$$

Which of the following are true?

✓ I. $\lim_{x \rightarrow 1^+} h(x)$ exists

✓ II. $\lim_{x \rightarrow 1^-} h(x)$ exists

✓ III. $\lim_{x \rightarrow 1} h(x)$ exists

✗ IV. $h(x)$ is continuous at $x = 1$

(A) I only

(B) I and II only

(C) I, II, and III only

(D) IV only

(E) I, II, III, and IV

I. $\lim_{x \rightarrow 1^+} h(x) = \lim_{x \rightarrow 1^+} \frac{1}{x} = \frac{1}{1} = 1$ ✓

II. $\lim_{x \rightarrow 1^-} h(x) = \lim_{x \rightarrow 1^-} x = 1$ ✓

III. $\lim_{x \rightarrow 1^-} h(x) = \lim_{x \rightarrow 1^+} h(x) = 1$, so $\lim_{x \rightarrow 1} h(x) = 1$ ✓

IV. $h(1)$

not defined,
so h not
continuous
at $x=1$

11. Evaluate the following limit:

[1]

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2}}{x}$$

(A) $+\infty$ (B) $-\infty$

(C) 0

(D) 1

(E) -1

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2}}{x} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2}}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2}}{1} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow \infty} \sqrt{x^2 + 2} \cdot \frac{1}{\sqrt{x^2}}$$

$$= \lim_{x \rightarrow \infty} \sqrt{\frac{x^2 + 2}{x^2}} = \lim_{x \rightarrow \infty} \sqrt{1 + \frac{2}{x^2}}$$

$$= \sqrt{1 + 0} = 1$$

12. The function $f(x) = \frac{x^2 + 1}{x^3 + 8}$ has which of the following?

[1]

(A) no vertical or horizontal asymptotes

(B) 1 vertical asymptote and 1 horizontal asymptote

(C) 2 vertical asymptotes and 1 horizontal asymptote

(D) 1 vertical asymptote and 2 horizontal asymptotes

(E) 1 vertical asymptote and no horizontal asymptotes

• Vertical: $x^3 + 8 = 0 \rightarrow x^3 = -8 \rightarrow x = -2$ (and $x^2 + 1 = 5 \neq 0$ at $x = -2$, so Vert. asymptote there)

→ 1 vertical

• Horizontal: $\lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{x^2 + 1}{x^3 + 8} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$$= \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{x} + \frac{1}{x^3}}{1 + \frac{8}{x^3}} = \frac{0+0}{1+0} = 0 \rightarrow \text{horiz. asym. } y=0$$

→ 1 horizontal

13. For what value of the number k is the following function differentiable at $x = 0$?

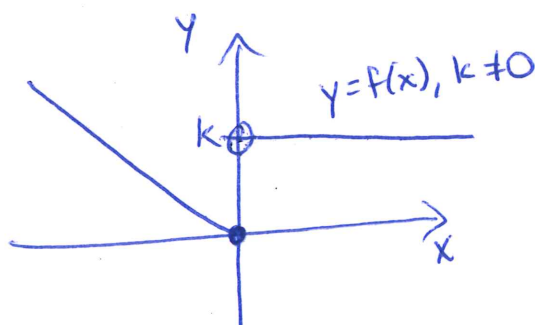
[1]

$$f(x) = \begin{cases} -x & x \leq 0 \\ k & x > 0 \end{cases}$$

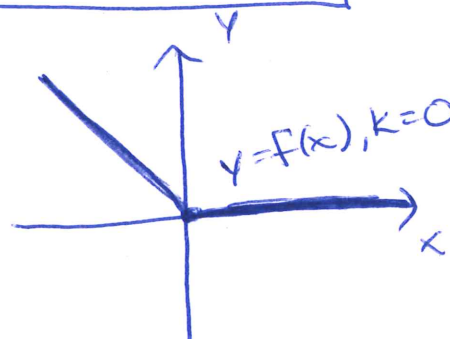
(A) -2 (B) -1 (C) 0

(D) 1

(E) No value of k makes this function differentiable at $x = 0$



not continuous at $x=0$,
so not
differentiable



not locally linear
at $x=0$, so not
differentiable

14. If $f(x) = 3x^{10}$, then $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$ is which of the following? [1]

- (A) $f'(x)$ (B) $f'(1)$ (C) Does not exist
(D) 0 (E) None of the above

$$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \underline{f'(1)} \text{ by limit def'n of derivative}$$

$$\left(f'(x) = 3 \cdot 10 x^9 = 30x^9, \text{ so } \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = f'(1) = 30 \cdot 1^9 = \underline{\underline{30}} \right)$$

15. If we want to calculate the derivative $f'(x)$ of $f(x) = 3x + 4$ using the limit definition of the derivative which of the following limits do we need to evaluate and to what does the limit evaluate? [1]

(A) $\lim_{h \rightarrow 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 3$

(B) $\lim_{h \rightarrow 0} \frac{3(x+h) + 4 - (3x+4)}{h} = 0$

(C) $\lim_{h \rightarrow 0} \frac{3h + 4 - (3x+4)}{h} = 3x + 3$

(D) $\lim_{h \rightarrow 0} \frac{3(x+h) + 4 - (3h+4)}{h} = 3$

(E) None of the above.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h) + 4 - (3x+4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x + 3h + 4 - 3x - 4}{h} \\ &= \lim_{h \rightarrow 0} \frac{3h}{h} = \lim_{h \rightarrow 0} 3 = \underline{\underline{3}} \end{aligned}$$

16. Below is the graph of the derivative $g'(x)$ of a function $g(x)$.

[1]

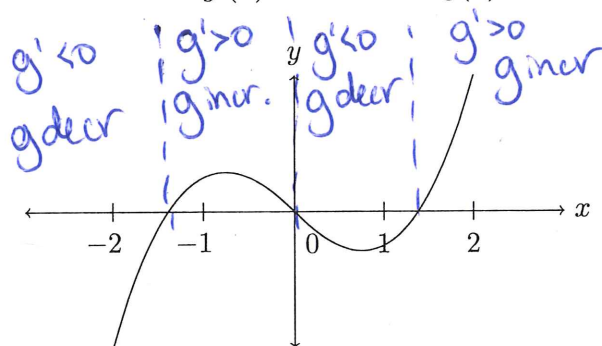
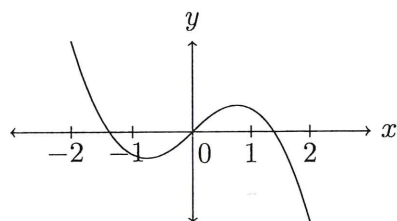


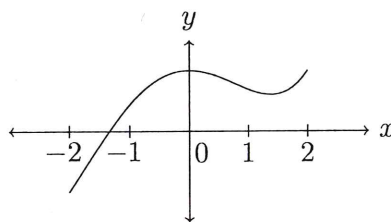
Figure 1: Graph of $g'(x)$.

Which of the following is a possible graph of $g(x)$?

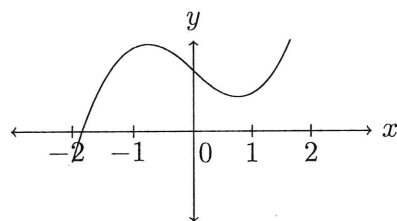
(A)



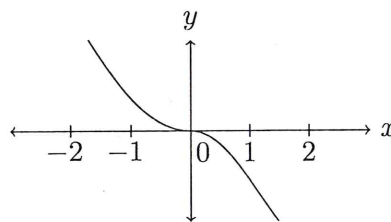
(B)



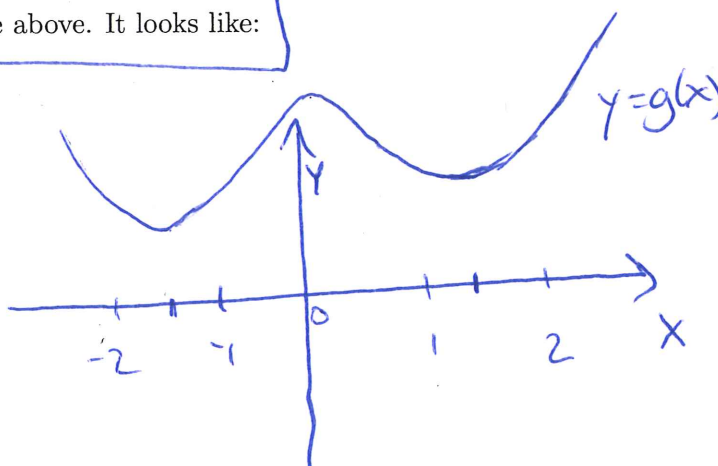
(C)



(D)



(E) None of the above. It looks like:



17. If $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ for $x > 0$, then $f'(4)$ is which of the following?

[1]

(A) $\frac{5}{4}$

(D) $\frac{3}{4}$

(C) $\frac{3}{16}$

(B) $\frac{255}{32}$

(E) $\frac{257}{32}$

$$f(x) = x^{1/2} + x^{-1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} + (-\frac{1}{2})x^{-3/2}$$

$$= \frac{1}{2} \left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}} \right)$$

$$= \frac{1}{2} \cdot \frac{x-1}{x\sqrt{x}}$$

$$f'(4) = \frac{1}{2} \cdot \frac{4-1}{4\sqrt{4}} = \underline{\underline{\frac{3}{16}}}$$