

Name: _____

Discussion Section: _____

Solutions should show all of your work, not just a single final answer.

2.5: Continuity

1. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 1, \\ a & \text{if } x = 1, \\ x - 1 & \text{if } x > 1. \end{cases}$$

(a) Determine the value of a for which $f(x)$ is continuous from the left at 1.

(b) Determine the value of a for which $f(x)$ is continuous from the right at 1.

(c) Is there a value of a for which $f(x)$ is continuous at 1? Explain.

2. Use the intermediate value theorem to show that there is a solution to $x - \sqrt{x} - \ln x = 0$ on the interval $(2, 3)$. Clearly explain your reasoning.

3. Let

$$f(x) = \begin{cases} 2 - kx & \text{if } x < 1, \\ k + x & \text{if } x > 1 \end{cases}$$

with the value of $f(1)$ to be determined.

(a) Compute $\lim_{x \rightarrow 1^-} f(x)$ in terms of k .

(b) Compute $\lim_{x \rightarrow 1^+} f(x)$ in terms of k .

(c) Find the values of k and $f(1)$ that make $f(x)$ continuous at $x = 1$.

(d) Using the choice of k and $f(1)$ in part (c), make a graph of $y = f(x)$ for $0 \leq x \leq 2$.

4. The function $f(x)$ is continuous on the interval $(-3, 4)$. If we know that $f(-1) = 4$ and $f(3) = 7$, what can we say about the outputs of $f(x)$, i.e. what values does f definitely take and/or not take?

5. T/F (with justification) The function

$$f(x) = \begin{cases} \sin x & \text{if } x \leq 0, \\ 1 + \cos x & \text{if } x > 0 \end{cases}$$

has a jump discontinuity at $x = 0$.

6. T/F (with justification) A function that is continuous at a point has to be defined at the point.

7. T/F (with justification) A function that is discontinuous at a point can't be defined at the point.

2.6: Limits at Infinity and Horizontal Asymptotes

8. Find the limit in each case or explain why it does not exist (and if it is $\pm\infty$).

(a) $\lim_{x \rightarrow \infty} \frac{2x + 3}{6x - 7}$

(b) $\lim_{x \rightarrow -\infty} \frac{x^3}{\sqrt{6x^4 - 1}}$

(c) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 3x} - x$

(d) $\lim_{x \rightarrow \infty} \frac{100000x}{x^3 + x}$

$$(e) \lim_{x \rightarrow \infty} \frac{\sqrt{16x^4 + 7x}}{8x^2 + 5}$$

$$(f) \lim_{x \rightarrow -\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$$

$$(g) \lim_{x \rightarrow \infty} \sqrt{x} + \sin x$$

$$(h) \lim_{x \rightarrow \infty} \frac{1}{x} + \sin x$$

9. Let $f(x) = \frac{\sqrt{4x^6 + 5}}{x^3 - 1}$.

(a) Compute $\lim_{x \rightarrow \infty} f(x)$.

(b) Compute $\lim_{x \rightarrow -\infty} f(x)$.

(c) What are the horizontal asymptotes of the graph of $y = f(x)$?

(d) What is the vertical asymptote of the graph of $y = f(x)$?

10. T/F (with justification) The graph of the function $y(x) = 3 + 6e^{-kx}$, with k a positive constant, has a horizontal asymptote $y = 6$.

11. T/F (with justification) If the continuous function $f(x)$ has domain $(-\infty, +\infty)$, then either $\lim_{x \rightarrow \infty} f(x)$ exists or $\lim_{x \rightarrow \infty} f(x)$ is $\pm\infty$.